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GRADE 12/*GRAAD 12*

MATHEMATICS/*WISKUNDE*

MEMORANDUM P2/*V2*

SEPTEMBER 2023

MARKS/*PUNTE*: 150

This memorandum consists of 19 pages/*Hierdie nasienriglyne bestaan uit 19 bladsye.*

QUESTION/VRAAG 1

1.1	<p style="text-align: center;">PREDICTION OF FINAL MARKS</p>	<p>✓✓✓✓ all dots correct/<i>alle punte korrek</i> ✓✓✓✓ 9 – 11 dots correct/<i>punte korrek</i> ✓✓✓ 6 – 8 dots correct/<i>punte korrek</i> ✓✓ 1 – 6 dots correct/<i>punte korrek</i></p> <p style="text-align: right;">(4)</p>
1.2	$r = 0,94$	<p>✓✓ answer/<i>antw</i></p> <p style="text-align: right;">(2)</p>
1.3	The strong correlation between the SBA mark and the final mark implies that the points lie close to the least squares regression line. Hence the prediction is reliable.	<p>✓ reason/<i>rede</i> ✓ conclusion/<i>konklusie</i></p> <p style="text-align: right;">(2)</p>
1.4	$a = 14,49$ $b = 0,86$ $y = 14,49 + 0,86x$	<p>✓ value/<i>waarde a</i> ✓ value/<i>waarde b</i> ✓ equation/<i>vgl</i></p> <p style="text-align: right;">(3)</p>
1.5	$y = 14,49 + 0,86(66)$ $y = 71,25\%$ OR/OF $y = 71,18\%$	<p>✓ subst/<i>vervang</i> ✓ answer/<i>antw</i></p> <p style="text-align: right;">(2)</p> <p>✓✓ answer/<i>antw</i></p> <p style="text-align: right;">(2)</p>

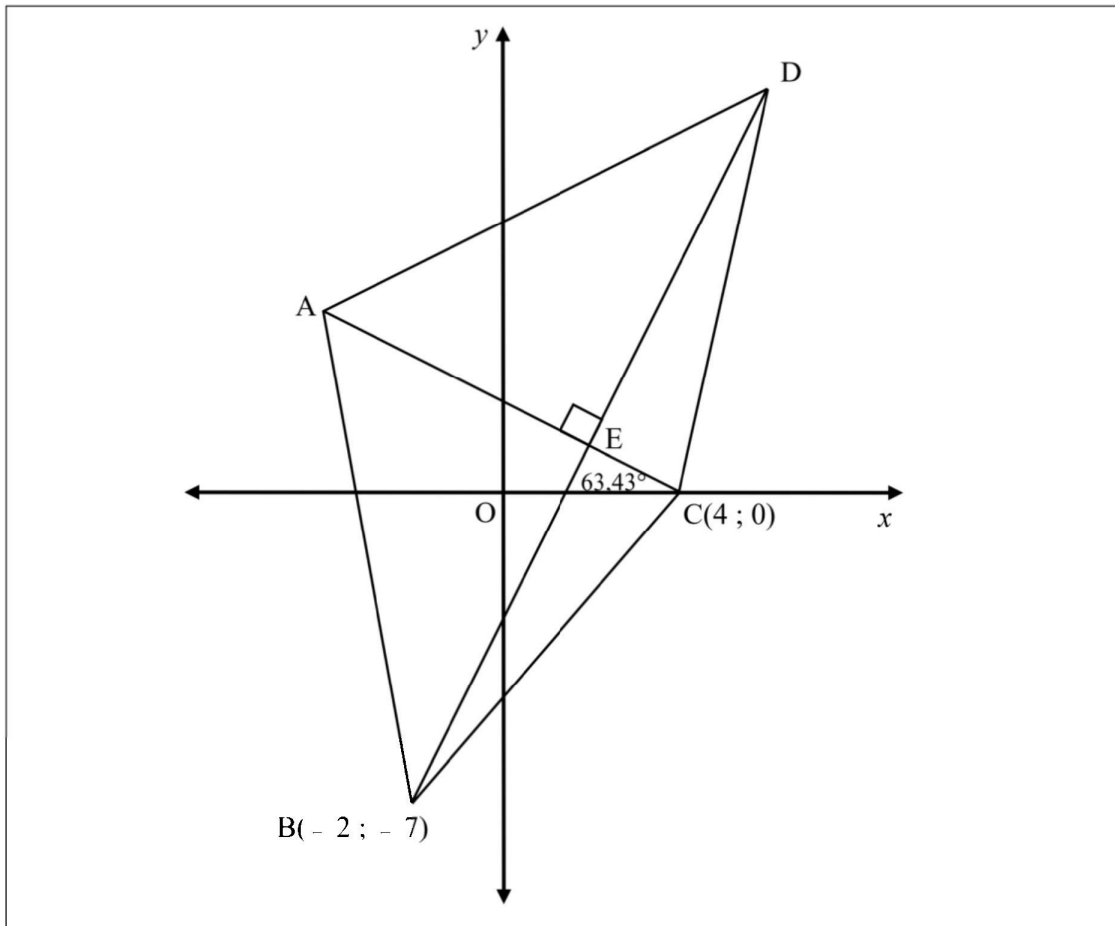
[13]

QUESTION/VRAAG 2

2.1.1	$\frac{3+4+4+6+10+12+12+4+y}{9} = 7$ $\frac{55+y}{9} = 7$ $55+y = 63$ $y = 8$	$\checkmark \frac{55+y}{9} = 7$ $\checkmark \text{value of/waarde } y$ (2)
2.1.2	Median = 6	$\checkmark 6$ (1)
2.2.1	$\bar{x} = \frac{3+4+4+4+6+8+10+12+12+7-n+7+n}{11}$ $\bar{x} = \frac{77}{11}$ $\bar{x} = 7$ <p>OR/OF</p> $\bar{x} = \frac{63+7-n+7+n}{11}$ $\bar{x} = \frac{77}{11}$ $\bar{x} = 7$	$\checkmark 77$ $\checkmark 7$ (2) $\checkmark 77$ $\checkmark 7$ (2)
2.2.2	$\bar{x} - \sigma_x = 3$ $7 - \sigma_x = 3$ $\sigma_x = 4$ <p>OR/OF</p> $\bar{x} + \sigma_x = 11$ $7 + \sigma_x = 11$ $\sigma_x = 4$	$\checkmark \text{equation/vgl}$ $\checkmark \text{answer/antw}$ (2) $\checkmark \text{equation/vgl}$ $\checkmark \text{answer/antw}$ (2)

[7]

QUESTION/VRAAG 3

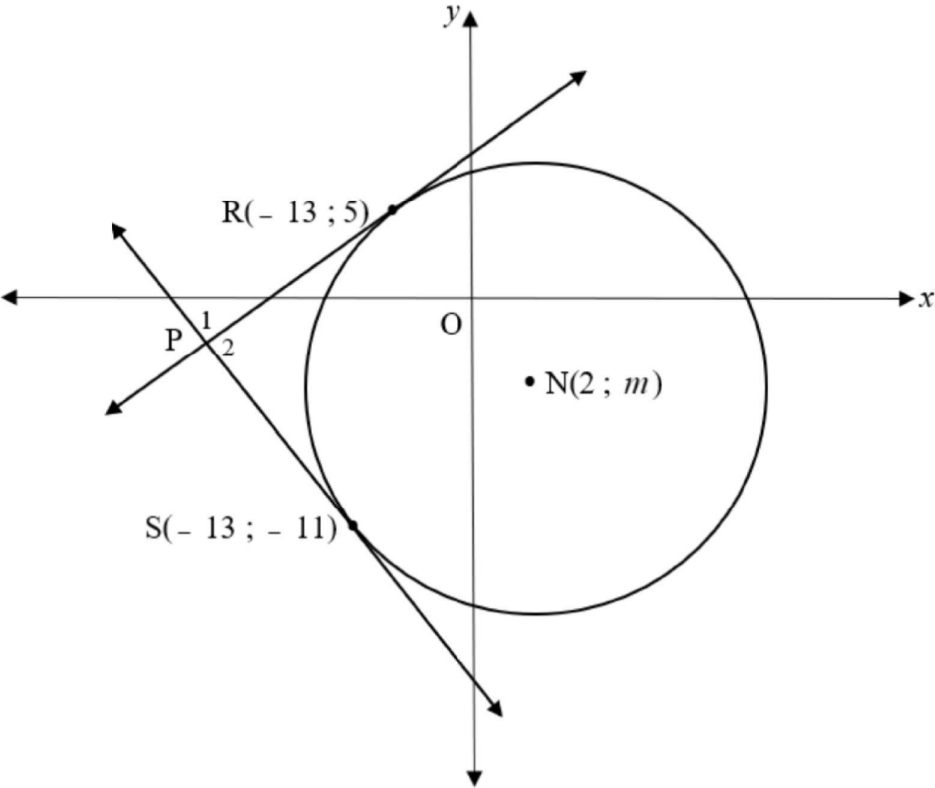


3.1	$y - y_1 = m(x - x_1)$ $y - (-7) = 2(x - (-2))$ $y + 7 = 2(x + 2)$ $BD: y = 2x - 3$ <p>OF/OR</p> $y = mx + c$ $-7 = 2(-2) + c$ $-7 = -4 + c$ $c = -3$ $BD: y = 2x - 3$	$m(AC) = -\frac{1}{2}$ $m(BD) = 2$ $m(AC) = -\frac{1}{2}$ $m(BD) = 2$	$\checkmark m(AC) = -\frac{1}{2}$ $\checkmark m(BD) = 2$ $\checkmark \text{subt } m \text{ and point B}$ $\checkmark \text{verv } m \text{ en punt B}$ $\checkmark \text{answer/antw}$ <p style="text-align: right;">(4)</p> $\checkmark m(AC) = -\frac{1}{2}$ $\checkmark m(BD) = 2$ $\checkmark \text{subt } m \text{ and point B}$ $\checkmark \text{verv } m \text{ en punt B}$ $\checkmark \text{answer/antw}$ <p style="text-align: right;">(4)</p>
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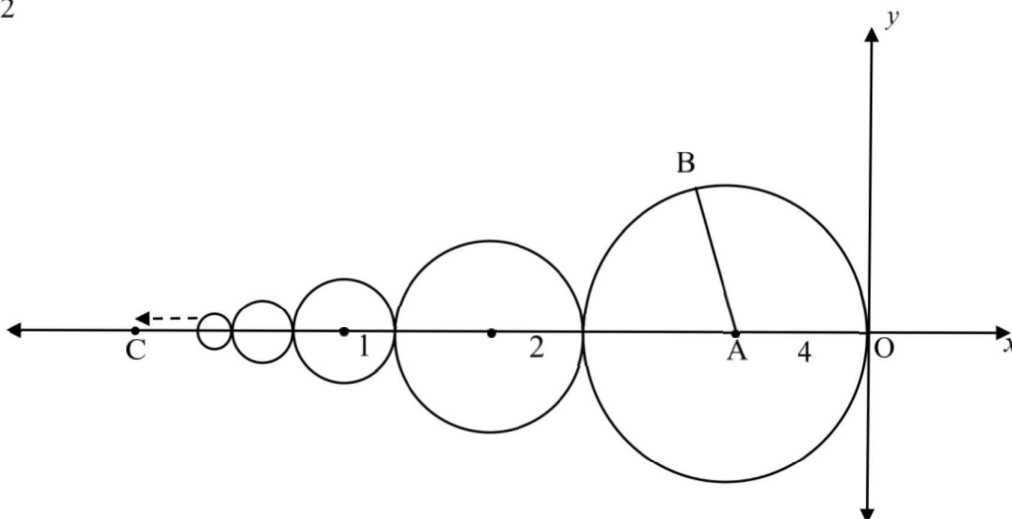
3.2	$-\frac{1}{2}x + 2 = 2x - 3$ $-x + 4 = 4x - 6$ $-5x = -10$ $x = 2$ subst into BD: $y = 2(2) - 3$ $y = 1$ $E(2; 1)$	✓ equating/vgl ✓ value of/waarde x ✓ value of/waarde y (3)
3.3	$A(-4; 4)$ through translation OR/OF Find a point P so that CE = EP $P(0; 2)$ Then CP = PA $A(-4; 4)$	✓ value of/waarde x ✓ value of/waarde y (2) ✓ P(0; 2) ✓ A(-4; 4) (2)
3.4	$AC = \sqrt{(-4-4)^2 + (4-0)^2} = 4\sqrt{5}$ $BE = \sqrt{(-2-2)^2 + (-7-1)^2} = 4\sqrt{5}$ Area of kite = $2 \times$ area of $\triangle ABC$ $= 2 \times \left(\frac{1}{2} \times 4\sqrt{5} \times 4\sqrt{5} \right)$ $= 80$ Enlarged by scale factor 2: $80 \times 4 = 320$ sq unit OR/OF $AC = \sqrt{(-4-4)^2 + (4-0)^2} = 4\sqrt{5}$ $\therefore PR = 8\sqrt{5}$ $BE = \sqrt{(-2-2)^2 + (-7-1)^2} = 4\sqrt{5}$ $\therefore QS = 8\sqrt{5}$ Area of kite = $2 \times$ area of $\triangle PQR$ $= 2 \times \left(\frac{1}{2} \times 8\sqrt{5} \times 8\sqrt{5} \right)$ $= 320$	✓ length/lengte AC ✓ length/lengte BE ✓ method/metode ✓ 80 ✓ answer/antw (5) ✓ length/lengte AC ✓ length/lengte PR ✓ length/lengte QS ✓ method/metode ✓ answer/antw (5)

[14]

QUESTION/VRAAG 4

4.1		
4.1.1 (a)	$(-13-2)^2 + (5-m)^2 = 17^2$ $225 + 25 - 10m + m^2 = 289$ $m^2 - 10m - 39 = 0$ $(m-13)(m+3) = 0$ $m = 13 \text{ or } m = -3$ <p>NA</p> <p>OR/OF</p> $(-13-2)^2 + (-11-m)^2 = 17^2$ $225 + (-11-m)^2 = 289$ $(-11-m)^2 = 64$ $-11-m = \pm 8$ $m = -19 \text{ or } m = -3$ <p>NA</p>	<p>✓ subst R and N/verv R en N</p> <p>✓ simplify/vereenv</p> <p>✓ std form/std vorm</p> <p>✓ value/value m</p> <p>(4)</p> <p>✓ subst S and N/verv S en N</p> <p>✓ simplify/vereenv</p> <p>✓ std form/std vorm</p> <p>✓ value/waarde m</p> <p>(4)</p>

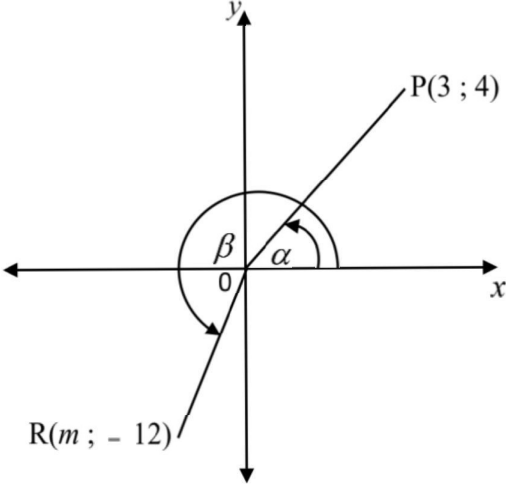
	<p>OR/OF</p> $(-13-2)^2 + (-11-m)^2 = (-13-2)^2 + (5-m)^2$ $225 + m^2 + 22m + 121 = 225 + m^2 - 10m + 25$ $32m = -96$ $m = -3$ <p>OR/OF</p> <p>RS \perp y-axis $\therefore x = -13$ same x-values Draw line NK \perp RS \therefore NK bisect RS at $(-13; -3)$ line from centre of circle \perp to chord $\therefore N(2; -3)$</p>	<p>✓✓ equating ✓ simplify/vereenv ✓ value/waarde m</p> <p>(4)</p> <p>✓ use line $x = -13$ ✓ Use NK ✓ midpt of NK ✓ $y = m = -3$</p> <p>(4)</p>
4.1.1 (b)	$(x-2)^2 + (y+3)^2 = 289$	<p>✓ answer/antw</p> <p>(1)</p>
4.1.2 (a)	$m(\text{NR}) = \frac{-3-5}{2-(-13)}$ $m(\text{NR}) = \frac{-8}{15}$	<p>✓ subst in gradient formula/verv in gradiënt formule ✓ gradient of NR</p> <p>(2)</p>
4.1.2 (b)	$m(\text{NS}) = \frac{-3-(-11)}{2-(-13)}$ $m(\text{NS}) = \frac{8}{15}$	<p>✓ gradient of NS/ gradiënt van NS</p> <p>(1)</p>
4.1.3	<p>NR \perp PR NS \perp PS</p> $\therefore m(\text{PR}) = \frac{15}{8} \qquad \therefore m(\text{PS}) = -\frac{15}{8}$ $\tan \alpha = \frac{15}{8} \qquad \tan \beta = -\frac{15}{8}$ $\alpha = 61,93^\circ \qquad \beta = 180^\circ - 61,93^\circ$ $\qquad \qquad \qquad \beta = 118,07^\circ$ $\hat{P}_1 = 118,07^\circ - 61,93^\circ = 56,14^\circ$ $\therefore \hat{P}_2 = 180^\circ - 56,14^\circ = 123,86^\circ$	<p>✓ $\tan \alpha$ def ✓ value/waarde α ✓ $\tan \beta$ def ✓ value/waarde β</p> <p>✓ method/metode ✓ answer/antw</p> <p>(6)</p>

	<p>OF/OR</p> <p>Inclination of NS = $28,07^\circ$ Inclination of NR = $151,93^\circ$ $R\hat{N}S = 28,07^\circ + 28,07^\circ$ $R\hat{N}S = 56,14^\circ$ NRPS is cyclic quadrilateral Opp \angle's suppl $\therefore \hat{P}_2 = 123,86^\circ$</p>	<p>$\checkmark 28,07^\circ$ $\checkmark 151,93^\circ$ $\checkmark R\hat{N}S = 2 \times 28,07^\circ$ $\checkmark 56,14^\circ$ \checkmark Opp \angle's cyclic quad/teenoorst \angle'e kvh \checkmark answer/antw</p> <p>(6)</p>
<p>4.1.4</p>	<p>Reflection about x-axis: $(2; -3) \rightarrow (2; 3)$ Shift 2 units up: $(2; 3) \rightarrow (2; 5)$ Circle M: $(x-2)^2 + (y-5)^2 = 289$</p>	<p>$\checkmark \checkmark$ equation/vgl</p> <p>(2)</p>
<p>4.2</p> 		
<p>4.2.1</p>	<p>Diameter = $8 + 4 + 2 + 1 \dots =$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8}{1-\frac{1}{2}}$ $S_\infty = 16$ $\therefore OC = 16$</p>	<p>\checkmark sum of diameters/ som middellyne \checkmark subst in sum formula/verv in som formule</p> <p>(2)</p>
<p>4.2.2</p>	<p>90°, radius \perp tangent</p>	<p>\checkmark answer/antw \checkmark reason/rede</p> <p>(2)</p>

4.2.3	$AC = 16 - 4 = 12$ $AB = 4$ $BC^2 = AC^2 - AB^2$ (Pyth) $BC^2 = 12^2 - 4^2$ $BC^2 = 128$ $BC = 8\sqrt{2}$ $\therefore \tan C = \frac{4}{8\sqrt{2}} = \frac{\sqrt{2}}{4}$	$\checkmark AC$ $\checkmark AB$ $\checkmark BC$ $\checkmark \tan \text{ratio}$ (4)
4.2.4	$y - y_1 = m(x - x_1)$ $y - 0 = \frac{\sqrt{2}}{4}(x - (-16))$ $y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$ OR/OF $y = \frac{\sqrt{2}}{4}x + c$ $0 = \frac{\sqrt{2}}{4}(-16) + c$ $c = 4\sqrt{2}$ $\therefore y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$	$\checkmark \text{subst/verv } m$ $\checkmark \text{subst point/verv}$ <i>punt</i> $\checkmark \text{equation/vgl}$ (3) $\checkmark \text{subst/verv } m$ $\checkmark \text{subst point/verv}$ <i>punt</i> $\checkmark \text{equation/vgl}$ (3)

[27]

QUESTION/VRAAG 5

5.1		
5.1.1	$\tan \alpha = \frac{4}{3}$	✓ answer/antw (1)
5.1.2	$\begin{aligned} \sin(90^\circ + \alpha) \\ &= \cos \alpha \\ &= \frac{3}{5} \end{aligned}$	✓ reduction/reduksie ✓ $r = 5$ ✓ answer/antw (3)
5.1.3	$\begin{aligned} 12 + 13 \sin \beta &= 0 \\ \sin \beta &= -\frac{12}{13} \\ m^2 &= 13^2 - (-12^2) \quad (\text{Pyth}) \\ m^2 &= 25 \\ m &= \pm 5 \\ \therefore m &= -5 \end{aligned}$	✓ std form/std vorm ✓ subst into Pyth ✓ simpl/vereenv ✓ answer/antw (4)
5.1.4	$\begin{aligned} \cos(\alpha + \beta) \\ &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) \\ &= -\frac{3}{13} + \frac{48}{65} \\ &= \frac{33}{65} \end{aligned}$	✓ expansion/uitbrei ✓ subst/vervang ✓ answer/antw (3)

5.2.1	$\sqrt{4^{\sin 150^\circ} \cdot 2^{3 \tan 225^\circ}}$ $= \sqrt{(2^2)^{\sin 30^\circ} \cdot 2^{3 \tan 45^\circ}}$ $= \sqrt{2^{2(\frac{1}{2})} \cdot 2^{3(1)}}$ $= \sqrt{2 \cdot 2^3}$ $= \sqrt{16}$ $= 4$	<p>✓ $\sin 30^\circ$ ✓ $\tan 45^\circ$ ✓ special values/ <i>spesiale waardes</i></p> <p>✓ simpl/vereenv</p> <p>✓ answer/antw</p> <p>(5)</p>
5.2.2	$\frac{\tan(180^\circ + x) \cos x}{\sin(180^\circ + x) \cos x - \cos(540^\circ + x) \cos(90^\circ + x)}$ $= \frac{(\tan x)(\cos x)}{(-\sin x)(\cos x) - (-\cos x)(-\sin x)}$ $= \frac{\frac{\sin x}{\cos x} \cdot \cos x}{-\sin x \cos x - \cos x \sin x}$ $= \frac{\sin x}{-2 \sin x \cos x}$ $= -\frac{1}{2 \cos x}$	<p>✓ $\tan x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$</p> <p>✓ answer/antw</p> <p>(6)</p>
5.3	$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2 \sin^2 x) - \sin x}{2 \sin x \cos x - \cos x}$ $= \frac{1 - 1 + 2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x}$ $= \frac{2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x}$ $= \frac{\sin x(2 \sin x - 1)}{\cos x(2 \sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	<p>✓ $\cos 2x$ expansion/ <i>uitbrei</i> ✓ $\sin 2x$ expansion/ <i>uitbrei</i></p> <p>✓ simpl/vereenv</p> <p>✓ factors/faktore</p> <p>(4)</p>

5.4	$\sin P \sin Q - \cos P \cos Q = \frac{1}{2}$ $\therefore \cos P \cos Q - \sin P \sin Q = -\frac{1}{2}$ $\cos(P+Q) = -\frac{1}{2}$ $P+Q = 180^\circ - 60^\circ$ $P+Q = 120^\circ \dots\dots\dots 1$ $\sin(P-Q) = \frac{1}{2}$ $P-Q = 30^\circ \dots\dots\dots 2$ $\therefore 2P = 150^\circ$ $P = 75^\circ$ $Q = 45^\circ$	<p>✓ rearrange terms/ herrangskik terme</p> <p>✓ cos identity/ identiteit</p> <p>✓ 2nd quadrant/ kwadrant</p> <p>✓ equation/vgl</p> <p>✓ 1st quadrant/ kwadrant</p> <p>✓ 75°</p> <p>✓ 45°</p> <p>(7)</p>
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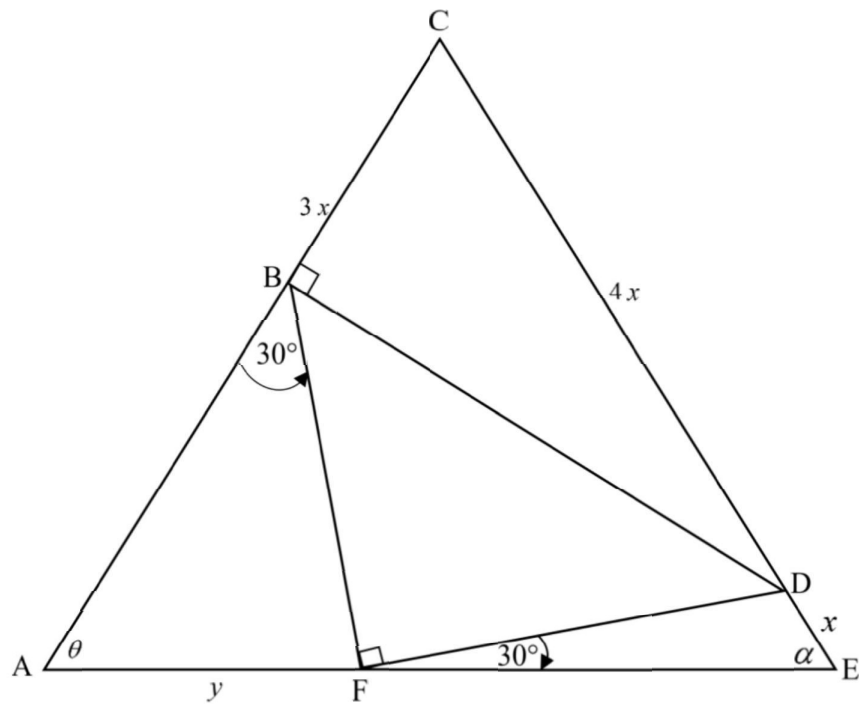
[33]

QUESTION/VRAAG 6

6.1	$a = 2$ and $b = 1$	<p>✓ value/waarde a</p> <p>✓ value/waarde b</p> <p>(2)</p>
6.2.1	$2 \sin x - \cos x = 0$ $2 \sin x = \cos x$ $\frac{\sin x}{\cos x} = \frac{1}{2}$ $\tan x = \frac{1}{2}$ $x = 26,57^\circ$	<p>✓ $\tan x = \frac{1}{2}$</p> <p>✓ answer/antw</p> <p>(2)</p>
6.2.2	$x \in [0^\circ ; 180^\circ]$	<p>✓ interval</p> <p>✓ notation/notasie</p> <p>(2)</p>
6.2.3	$y = 2^{2f(x)-1}$ $y \in \left[-\frac{1}{2} ; 7 \right]$	<p>✓ ✓ answer/antw</p> <p>(2)</p>

[8]

QUESTION/VRAAG 7

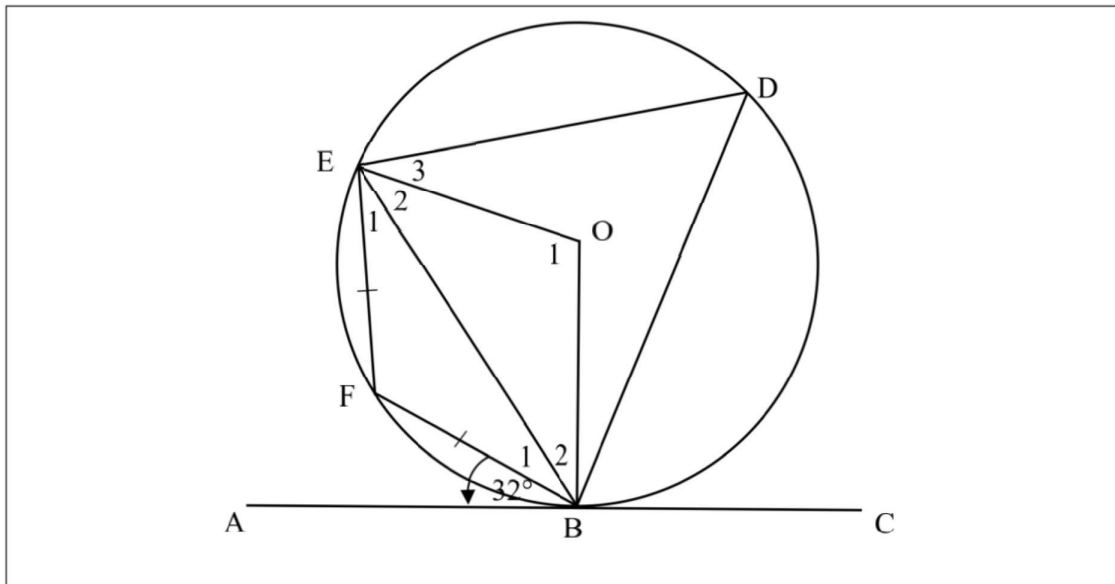


7.1	$\text{In } \triangle ABF: \frac{BF}{\sin A} = \frac{AF}{\sin B}$ $\frac{BF}{\sin \theta} = \frac{y}{\sin 30^\circ}$ $BF = \frac{y \sin \theta}{\frac{1}{2}}$ $BF = 2y \sin \theta$	<p>✓ correct subst in sine rule/korrekte verv in sinreel ✓ subst special value/ verv spasiale waarde ✓ answer/antw</p> <p style="text-align: right;">(3)</p>
7.2	$\text{In } \triangle EDF: \frac{DF}{\sin E} = \frac{DE}{\sin E}$ $\frac{DF}{\sin \alpha} = \frac{x}{\sin 30^\circ}$ $DF = \frac{x \sin \alpha}{\frac{1}{2}}$ $DF = 2x \sin \alpha$	<p>✓ correct subst in sine rule/korrekte verv in sinreel</p> <p>✓ answer/antw</p> <p style="text-align: right;">(2)</p>
7.3	$BD^2 = BF^2 + FD^2 \quad (\text{Pyth})$ $BD^2 = (2y \sin \theta)^2 + (2x \sin \alpha)^2$ $BD^2 = 4y^2 \sin^2 \theta + 4x^2 \sin^2 \alpha$	<p>✓ subst/verv in Pyth</p> <p style="text-align: right;">(1)</p>

7.4	<p>In $\triangle BDC$: $BD^2 = (4x)^2 - (3x)^2$ $BD^2 = 16x^2 - 9x^2$ $BD^2 = 7x^2$</p> <p>$7x^2 = 4y^2 \sin^2 \theta + 4x^2 \sin^2 \alpha$ $7x^2 - 4x^2 \sin^2 \alpha = 4y^2 \sin^2 \theta$ $x^2(7 - 4\sin^2 \alpha) = 4y^2 \sin^2 \theta$ $x^2 = \frac{4y^2 \sin^2 \theta}{(7 - 4\sin^2 \alpha)}$ $x = \sqrt{\frac{4y^2 \sin^2 \theta}{7 - 4\sin^2 \alpha}}$</p> <p>OF/OR</p> <p>$CD^2 = BD^2 + BC^2$ $16x^2 = 4y^2 \sin^2 \theta + 4x^2 \sin^2 \alpha + 9x^2$ $16x^2 - 9x^2 - 4x^2 \sin^2 \alpha = 4y^2 \sin^2 \theta$ $7x^2 - 4x^2 \sin^2 \alpha = 4y^2 \sin^2 \theta$ $x^2(7 - 4\sin^2 \alpha) = 4y^2 \sin^2 \theta$ $x^2 = \frac{4y^2 \sin^2 \theta}{7 - 4\sin^2 \alpha}$ $x = \sqrt{\frac{4y^2 \sin^2 \theta}{7 - 4\sin^2 \alpha}}$</p>	<p>✓ $BD^2 = 7x^2$</p> <p>✓ equating</p> <p>✓ simpl/vereenv</p> <p>(3)</p> <p>✓ subst/verv in Pyth</p> <p>✓ simpl/vereenv</p> <p>✓ i.t.o x^2</p> <p>(3)</p>
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[9]

QUESTION/VRAAG 8

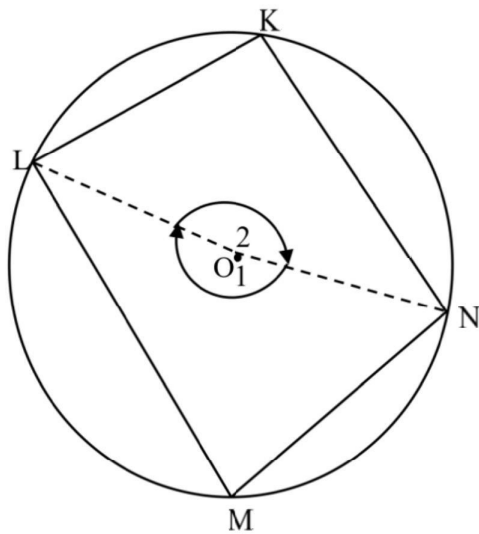


8.1	$\hat{A}BF = \hat{E}_1 = 32^\circ$	tan-chord theorem	\checkmark S \checkmark R (2)
8.2	$\hat{E}_1 = \hat{B}_1 = 32^\circ$ $\hat{F} = 116^\circ$	\angle 's opp equal sides sum \angle 's of Δ	\checkmark S/R \checkmark S/R (2)
8.3	$\hat{D} + \hat{F} = 180^\circ$ $\hat{D} = 64^\circ$	opp \angle 's of cyclic quad	\checkmark S \checkmark R (2)
8.4	$\hat{O}_1 = 2 \times \hat{D}$ $\hat{O}_1 = 128^\circ$	midpt $\angle = 2 \times$ circumf \angle	\checkmark S \checkmark R (2)
8.5	$\hat{E}_2 + \hat{O}_1 + \hat{B}_2 = 180^\circ$ $\hat{E}_2 = \hat{B}_2$ $2\hat{E}_2 + 128^\circ = 180^\circ$ $2\hat{E}_2 = 52^\circ$ $\hat{E}_2 = 26^\circ$ OR/OF $\hat{A}BF + \hat{B}_1 + \hat{B}_2 = 90^\circ$ $32^\circ + 32^\circ + \hat{B}_2 = 90^\circ$ $\hat{B}_2 = 26^\circ$ $\hat{B}_2 = \hat{E}_2 = 26^\circ$	sum \angle 's of Δ \angle 's opp equal sides rad \perp tangent \angle 's opp equal sides	\checkmark S/R \checkmark S/R \checkmark S/R (2)

[10]

QUESTION/VRAAG 9

9.1



Construction: Join LO and ON

$$\hat{O}_1 = 2 \times \hat{K} \quad \angle \text{at centre} = 2 \times \angle \text{at circumf}$$

$$\hat{O}_2 = 2 \times \hat{M} \quad \angle \text{at centre} = 2 \times \angle \text{at circumf}$$

$$\hat{O}_1 + \hat{O}_2 = 360^\circ \quad \text{revolution}$$

$$2\hat{M} + 2\hat{K} = 360^\circ \quad \text{equating}$$

$$\hat{M} + \hat{K} = 180^\circ$$

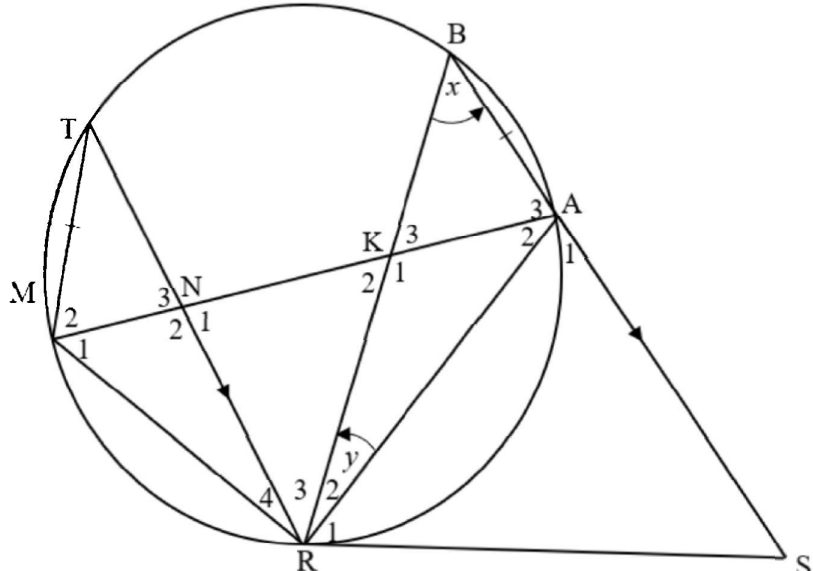
✓ constr/konstr

✓S ✓R

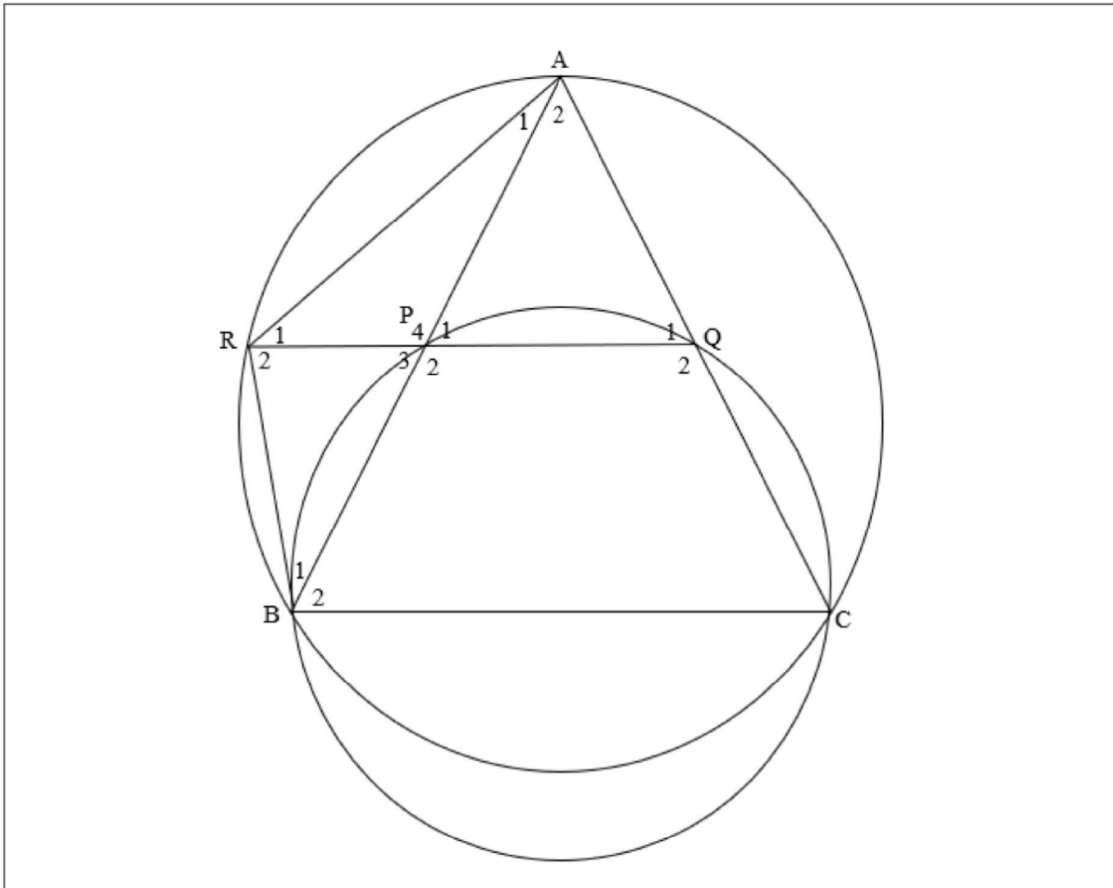
✓S/R

✓R

(5)

9.2		
9.2.1	$\hat{B} = \hat{M}_1 = x$ \angle 's in the same segment $\hat{B} = \hat{R}_3 = x$ alt \angle 's, $TR \parallel BS$ $\hat{B} = \hat{R}_1 = x$ tan-chord theorem	\checkmark S \checkmark R \checkmark S \checkmark R \checkmark S \checkmark R (6)
9.2.2	Equal chords equal circumf \angle 's	\checkmark R (1)
9.2.3(a)	$\hat{A}_1 = x + y$ ext \angle of Δ	\checkmark S/R (1)
9.2.3(b)	$\hat{N}_1 = x + y$ ext \angle of Δ	\checkmark S/R (1)
9.2.4	In ΔSAR and ΔKNR (i) $\hat{R}_1 = \hat{R}_3$ both = x (ii) $\hat{A}_1 = \hat{N}_1$ both = $x + y$ (iii) $\hat{K}_2 = \hat{S}$ sum \angle 's of Δ $\therefore \Delta SAR \parallel \Delta KNR \quad \angle \angle \angle$ OR/OF In ΔSAR and ΔKNR (i) $\hat{R}_1 = \hat{R}_3$ both = x (ii) $\hat{A}_1 = \hat{N}_1$ both = $x + y$ $\therefore \Delta SAR \parallel \Delta KNR \quad \angle \angle \angle$	\checkmark S \checkmark S \checkmark S (3) \checkmark S \checkmark S \checkmark R (3)
9.2.5	$\hat{K}_2 = \hat{S} \quad \Delta SAR \parallel \Delta KNR$ SAKR is a cyclic quad ext \angle of quadrilateral OR converse ext \angle of cyclic quad	\checkmark S \checkmark R (2)

QUESTION/VRAAG 10



10.1	$\hat{P}_1 = C$ $\hat{C} = 180^\circ - (\hat{R}_1 + \hat{R}_2)$ $\therefore \hat{P}_1 = 180^\circ - (\hat{R}_1 + \hat{R}_2)$ $\hat{R}_1 + \hat{R}_2 = 180^\circ - (\hat{A}_1 + \hat{B}_1) \quad \text{sum } \angle \text{'s of } \Delta$ $\therefore \hat{P}_1 = 180^\circ - [180^\circ - (\hat{A}_1 + \hat{B}_1)]$ $\therefore \hat{P}_1 = 180^\circ - 180^\circ + (\hat{A}_1 + \hat{B}_1)$ $\therefore \hat{P}_1 = \hat{A}_1 + \hat{B}_1$	<p>ext \angle of cyclic quad</p> <p>opp \angle's of cyclic quad</p> <p>✓S ✓R</p> <p>✓S ✓R</p> <p>✓method/metode</p> <p>(5)</p>
------	---	--

	<p>OR/OF</p> $\widehat{A}\widehat{R}\widehat{B} = 180^\circ - (\widehat{A}_1 + \widehat{B}_1)$ $\widehat{A}\widehat{R}\widehat{B} = 180^\circ - \widehat{C}$ $\therefore 180^\circ - (\widehat{A}_1 + \widehat{B}_1) = 180^\circ - \widehat{C}$ $\widehat{C} = \widehat{A}_1 + \widehat{B}_1$ $\widehat{C} = \widehat{P}_1$ $\therefore \widehat{P}_1 = \widehat{A}_1 + \widehat{B}_1$	<p>✓S ✓R</p> <p>✓S ✓R</p> <p>✓method/metode</p> <p>(5)</p>
10.2	<p>In $\triangle ARP$ and $\triangle ABR$</p> <p>(i) $\widehat{A} = \widehat{A}$ common \angle</p> <p>(ii) $\widehat{P}_1 = \widehat{A}_1 + \widehat{B}_1$ proven</p> $180^\circ - \widehat{P}_1 = 180^\circ - (\widehat{A}_1 + \widehat{B}_1)$ $\therefore \widehat{P}_4 = \widehat{R}_1 + \widehat{R}_2$ <p>(iii) $\widehat{R}_1 = \widehat{B}_1$ sum \angle's of \triangle</p> $\therefore \triangle ARP \parallel \triangle ABR$ $\frac{AR}{AB} = \frac{AP}{AR}$ $AR^2 = AB \cdot AP$ <p>OR/OF</p> <p>In $\triangle ARP$ and $\triangle ABR$</p> <p>(i) $\widehat{A} = \widehat{A}$ common \angle</p> <p>(ii) $\widehat{P}_1 = \widehat{A}_1 + \widehat{B}_1$ proven</p> $180^\circ - \widehat{P}_1 = 180^\circ - (\widehat{A}_1 + \widehat{B}_1)$ $\therefore \widehat{P}_4 = \widehat{R}_1 + \widehat{R}_2$ $\therefore \triangle ARP \parallel \triangle ABR$ $\frac{AR}{AB} = \frac{AP}{AR}$ $AR^2 = AB \cdot AP$	<p>✓ identify/ identifiseer \triangle's</p> <p>✓S</p> <p>✓method/metode</p> <p>✓R</p> <p>✓S</p> <p>(5)</p> <p>✓Identify/ identifiseer \triangle's</p> <p>✓S</p> <p>✓method/metode</p> <p>✓R</p> <p>✓S</p> <p>(5)</p>

(5)
[10]

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18 AUGUST 2023

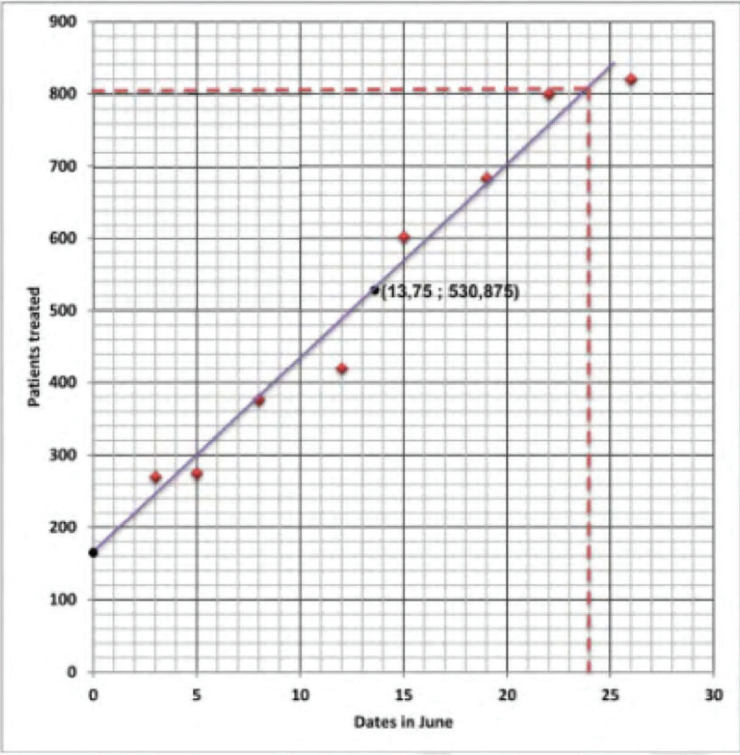
MARKS: 150

This marking guideline consists of 14 pages.

QUESTION 1

1.1	<table border="1"> <thead> <tr> <th>Speed (km/h)</th> <th>Frequency (f)</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td>$0 < x \leq 10$</td> <td>10</td> <td>10</td> </tr> <tr> <td>$10 < x \leq 20$</td> <td>20</td> <td>30</td> </tr> <tr> <td>$20 < x \leq 30$</td> <td>45</td> <td>75</td> </tr> <tr> <td>$30 < x \leq 40$</td> <td>72</td> <td>147</td> </tr> <tr> <td>$40 < x \leq 50$</td> <td>23</td> <td>170</td> </tr> </tbody> </table>	Speed (km/h)	Frequency (f)	Cumulative frequency	$0 < x \leq 10$	10	10	$10 < x \leq 20$	20	30	$20 < x \leq 30$	45	75	$30 < x \leq 40$	72	147	$40 < x \leq 50$	23	170	<ul style="list-style-type: none"> ✓ column 1 values ✓ column 2 values <p style="text-align: right;">(2)</p>
Speed (km/h)	Frequency (f)	Cumulative frequency																		
$0 < x \leq 10$	10	10																		
$10 < x \leq 20$	20	30																		
$20 < x \leq 30$	45	75																		
$30 < x \leq 40$	72	147																		
$40 < x \leq 50$	23	170																		
1.2 & 1.3		<ul style="list-style-type: none"> ✓ Grounding(0; 0) ✓ Maximum (50; 170) ✓ Curve <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ Q_1 ✓ Q_2 <p style="text-align: right;">(2)</p>																		
1.4		<ul style="list-style-type: none"> ✓ Q_3 <ul style="list-style-type: none"> ✓ Box <p style="text-align: right;">(2)</p>																		
1.5	164 (Accept 163 – 165)	<ul style="list-style-type: none"> ✓ Answer <p style="text-align: right;">(1)</p>																		
[10]																				

QUESTION 2

2.1		<p>✓ ✓ ✓ ALL points plotted correctly</p> <p>-1 for every 2 mistakes</p> <p>-1 if points are joined</p> <p>(3)</p>
2.2	$y = 26,88x + 161,24$	<p>✓ value of $a = 161,24$</p> <p>✓ value of $b = 26,88$</p> <p>✓ equation of the line (3)</p>
2.3	$y = 26,88(24) + 161,24$ $y = 806,36$ 806 people treated on 24th of June	<p>✓ substitute in formula</p> <p>✓ answer as Natural Number (2)</p>
2.4	see grid	<p>✓ straight line</p> <p>✓ graph passes through $(x ; y)$</p> <p>✓ graph passes through y-intercept from regression line formula (3)</p>
2.5	$r = 0,98$ This is a very strong positive, linear correlation. As the days increase the number of patients increase	<p>✓ ✓ correct answer</p> <p>✓ interpretation (3)</p>
2.6	$(\bar{x} - \sigma ; \bar{x} + \sigma)$ $(528,63 - 210,46 ; 528,63 + 210,46)$ $(318,17 ; 739,09)$ 4 patients.	<p>✓ 318,17</p> <p>✓ (739,09)</p> <p>✓ answer (3)</p>

QUESTION 3

3.1	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BC} = \frac{8 - 0}{4 - 5}$ $m_{BC} = -8$	<ul style="list-style-type: none"> ✓ correct substitution into the correct formula ✓ answer <p style="text-align: right;">(2)</p>
3.2	$AB = \sqrt{(-3 - 4)^2 + (k - 8)^2}$ $65 = 49 + k^2 - 16k + 64$ $k^2 - 16k + 48 = 0$ $(k - 4)(k - 12) = 0$ $k = 4 \text{ or } k = 12$ $k = 4$	<ul style="list-style-type: none"> ✓ substitute A and B into distance formula ✓ standard form ✓ factors ✓ $k = 4$ <p style="text-align: right;">(4)</p>
3.3	$m_{BD} = \frac{8 - (-4)}{4 - (-2)}$ $m_{BD} = 2$ $m_{AC} = \frac{4 - 0}{-3 - 5} = -\frac{1}{2}$ $m_{AC} \times m_{BD} = 2 \times -\frac{1}{2} = -1$ $AC \perp BD$	<ul style="list-style-type: none"> ✓ m_{BD} ✓ m_{AC} ✓ $m_{BD} \times m_{AC}$ <p style="text-align: right;">(3)</p>
3.4	<p><i>midpoint of AC = midpoint of DC</i></p> $\frac{x + (-3)}{2} = \frac{-2 + 5}{2} \text{ and } \frac{y + 4}{2} = \frac{-4 + 0}{2}$ $x = 6 \text{ or } y = -8$ $F(6; -8)$	<ul style="list-style-type: none"> ✓ x-coordinate ✓ y-coordinate
3.5	$m_{AD} = m_{BC} = -8$ $\tan \alpha = -8$ $\alpha = 180^\circ - \tan^{-1}(8)$ $\alpha = 97,13^\circ$ $\tan \beta = m_{BD} = 2$ $\beta = 63,43^\circ$ $O_1 = 63,43^\circ (\text{vert opp } \angle^s)$	<ul style="list-style-type: none"> ✓ m_{AD} ✓ $\tan \alpha = -8$ ✓ $\alpha = 97,13^\circ$ ✓ $\beta = 63,43^\circ$ ✓ $\alpha - \beta$ ✓ answer <p style="text-align: right;">(6)</p>

	$\hat{E}DO = \alpha - \beta$ $\hat{E}DO = 97,13^\circ - 63,43^\circ = 33,7^\circ$	
3.6	$AC = \sqrt{(5 - (-3))^2 + (0 - 4)^2} = \sqrt{80} = 4\sqrt{5}$ $DP = \sqrt{(-2 - 1)^2 + (-4 - 2)^2} = \sqrt{45} = 3\sqrt{5}$ $\text{Area of } \triangle ADC = \frac{1}{2} AC \times DP$ $\text{Area of } \triangle ADC = \frac{1}{2} (4\sqrt{5})(4\sqrt{5})$ $\text{Area of } \triangle ADC = 30 \text{ square units}$	<ul style="list-style-type: none"> ✓ length of AC ✓ length of DP ✓ correct substitution into formula ✓ answer <p style="text-align: right;">(4)</p>
		[08]

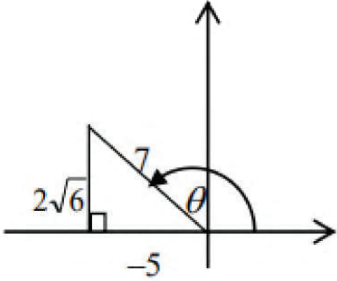
QUESTION 4

4		
4.1	$OB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $OB = \sqrt{(-4 - 0)^2 + (-6 - 0)^2}$ $OB = \sqrt{52}$ $OB = 2\sqrt{13}$ $r = \sqrt{13}$	<ul style="list-style-type: none"> ✓ substitution ✓ $OB = \sqrt{52}$ ✓ $r = \sqrt{13}$ <p style="text-align: right;">(3)</p>
4.2	$C\left(\frac{-4+0}{2}; \frac{-6+0}{2}\right)$ $C(-2; -3)$ $(x+2)^2 + (y+3)^2 = 13$	
4.3	Right-angled triangle, tangent perpendicular to the radius.	<ul style="list-style-type: none"> ✓ right angled Δ ✓ reason <p style="text-align: right;">(2)</p>
4.4	$m_{OB} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BO} = \frac{-6 - 0}{-4 - 0}$ $m_{BO} = \frac{3}{2}$ $m_{KL} = -\frac{2}{3}$ $y - y_1 = m(x - x_1)$ $y + 6 = -\frac{2}{3}(x + 4)$	<ul style="list-style-type: none"> ✓ m_{OB} ✓ m_{KL} ✓ substitution ✓ answer <p style="text-align: right;">(4)</p>

	$y = -\frac{2}{3}x - \frac{26}{3}$	
4.5	E(0;-6)	✓✓ answer (2)
4.6	$(x+2)^2 + (0+3)^2 = 13$ $F(-4;0)$ $m_{FC} = \frac{-3-0}{-2+4} = -\frac{3}{2}$ $m_{CE} = \frac{-3+6}{-2+0} = -\frac{3}{2}$ Points F, C and E are collinear EF is a diameter	✓ x_F ✓ y_F ✓ m_{FC} ✓ m_{CE} ✓ Points F, C and E are collinear (5)
		[20]

QUESTION 5

5.1.1	$\tan 58^\circ = m$ $x^2 + y^2 = r^2$ $1^2 + m^2 = r^2$ $\sqrt{1+m^2} = r$ $\sin 58^\circ = \frac{m}{\sqrt{1+m^2}}$ <div style="text-align: center;"> </div>	✓ $\sqrt{1+m^2} = r$ ✓ answer (2)
5.1.2	$\sin 296^\circ = -\sin 64^\circ$ $\sin 296^\circ = -\sin 2(32^\circ)$ $\sin 296^\circ = -2 \sin 32^\circ \times \cos 32^\circ$ $\sin 296^\circ = -2 \times \frac{m}{\sqrt{1+m^2}} \times \frac{1}{\sqrt{1+m^2}}$ $\sin 296^\circ = -\frac{2m}{1+m^2}$	✓ $-\sin 64^\circ$ ✓ substitution ✓ answer (3)
5.2.1	$5 \tan \theta + 2\sqrt{6} = 0$	

	 <p> $\tan \theta = -\frac{2\sqrt{6}}{5}$ $0^\circ < \theta < 270^\circ$ </p> <p> $r^2 = y^2 + x^2$ $r^2 = (-2\sqrt{6})^2 + (5)^2$ $r^2 = 49$ $r = 7$ $\sin \theta = \frac{2\sqrt{6}}{7}$ </p>	$\checkmark r = \frac{\sqrt{649}}{5}$ $\checkmark \sin \theta = \frac{2\sqrt{6}}{\sqrt{649}}$ $\frac{5}{5}$ (2)
5.2.2	$\cos \theta = -\frac{5}{7}$	\checkmark answer (1)
5.2.3	$\frac{14 \cos \theta + 7\sqrt{6} \sin \theta}{\cos(-240^\circ) \cdot \tan 225^\circ}$ $= \frac{14\left(-\frac{5}{7}\right) + 7\sqrt{6}\left(\frac{2\sqrt{6}}{7}\right)}{\cos(90^\circ + 30^\circ) \cdot \tan(180^\circ + 45^\circ)}$ $= \frac{-10 + 12}{-\sin 30^\circ \cdot \tan 45^\circ}$ $= \frac{2}{-\frac{1}{2} \times 1}$ $= -4$	$\checkmark -\sin 30^\circ$ $\checkmark \tan 45^\circ$ $\checkmark -\frac{1}{2} \times 1$ \checkmark answer (4)
		[15]

QUESTION 6

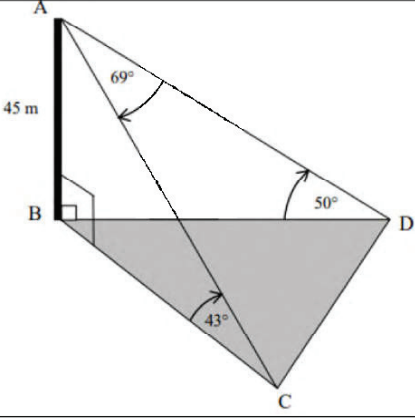
6.1	$\frac{\cos(180^\circ + x) \cdot \tan(360^\circ - x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ $= \frac{(-\cos x) \cdot (-\tan x) \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \frac{\cos x \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \cos^2 x + \sin^2 x$ $= 1$	<ul style="list-style-type: none"> ✓ $-\cos x$ ✓ $-\tan x$ ✓ $\cos^2 x$ ✓ $\sin x$ ✓ $\cos^2 x + \sin^2 x$ ✓ answer <p style="text-align: right;">(6)</p>
6.2.1	$\cos(A - B) - \cos(A + B)$ $= \cos A \cos B + \sin A \sin B - [\cos A \cos B - \sin A \sin B]$ $= \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$ $= 2\sin A \sin B$	<ul style="list-style-type: none"> ✓ ✓ ✓ <p style="text-align: right;">(3)</p>
6.2.2	$\cos 15^\circ - \cos 75^\circ = \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= 2\sin 45^\circ \cdot \sin 30^\circ$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad \text{or/ of} \quad 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{2}}{2} \quad \text{or/ of} \quad \frac{1}{\sqrt{2}}$ <p>OR</p> $\cos 15^\circ - \cos 75^\circ$ $= \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ - [\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ]$ $= 2\sin 45^\circ \sin 30^\circ$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad \text{or/ of} \quad 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{2}}{2} \quad \text{or/ of} \quad \frac{1}{\sqrt{2}}$	<ul style="list-style-type: none"> ✓ ✓ ✓ ✓ <p style="text-align: right;">(4)</p>

<p>6.3</p>	$AB^2 = (\cos \theta - 6)^2 + (\sin \theta - 7)^2$ $86 = \cos^2 \theta - 12\cos \theta + 36 + \sin^2 \theta - 14\sin \theta + 49$ $86 = 1 + 36 + 49 - 12\cos \theta - 14\sin \theta$ $0 = -12\cos \theta - 14\sin \theta$ $14\sin \theta = -12\cos \theta$ $\frac{\sin \theta}{\cos \theta} = \frac{-12}{14}$ $\tan \theta = -\frac{6}{7} / -0,86$	<p>✓ ✓ ✓ ✓</p> <p>(4)</p>

QUESTION 7

7.1		<ul style="list-style-type: none"> ✓ shape f ✓ intercepts ✓ turning point ✓ shape f ✓ intercepts ✓ turning point <p style="text-align: right;">(6)</p>
7.2.1	$x = -45^\circ$ or $x = 135^\circ$	✓ ✓ answer (2)
7.2.2	$x = 180^\circ$ or/of $x = -180^\circ$	✓ ✓ answer (2)
7.2.3	$y \in [-1; 1]$ or/of $-1 \leq y \leq 1$	✓ answer (1)
7.2.4	1	✓ answer (1)
7.2.5	$x \in (-180^\circ; -45^\circ)$ or/of $(0^\circ; 135^\circ)$ OR $-180^\circ < x < -45^\circ$ or/of $0^\circ < x < 135^\circ$	<ul style="list-style-type: none"> ✓ end points ✓ notation ✓ answer <p>(3)</p>
		[15]

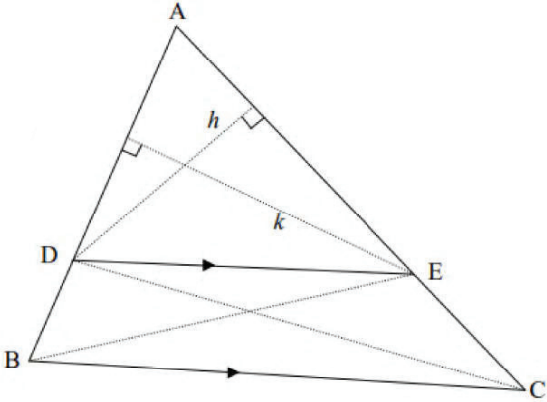
QUESTION 7

		
7.1	$\sin 43^\circ = \frac{45}{AC}$ $AC = 66m$ $\sin 43^\circ = \frac{45}{AC}$ $\sin 50^\circ = \frac{45}{AD}$ $AD = 58,74m$	$\checkmark \sin 43^\circ = \frac{45}{AC}$ $\checkmark AC$ $\checkmark \sin 43^\circ = \frac{45}{AC}$ $\checkmark AD$ <p style="text-align: right;">(4)</p>
7.2	$CD^2 = AC^2 + AD^2 - 2AC \cdot AD \cos 69^\circ$ $CD^2 = (66)^2 + (58.74)^2 - 2(66)(58.74)\cos 69^\circ$ $CD^2 = 5027,72$ $CD = 70.91m$	$\checkmark \text{ using cosine rule}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p>

QUESTION 8

8.1.1	$\hat{D}_1 = \hat{B}_2 = 30^\circ$ (alter angles DC parallel GB)	✓ S&R (1)
8.1.2	$\hat{B}_1 = \hat{C}_1 = 60^\circ$ (tan-chord theorem)	✓ S ✓ R (2)
8.1.3	$\hat{C}_2 = 90^\circ$ (sum of angles in triangle)	✓ S&R (1)
8.1.4	$\hat{DAB} + \hat{C}_2 = 180^\circ$ $\hat{DAB} = 90^\circ$	✓ S ✓ R (2)
8.2	Yes is diameter, converse angle in a semi-circle	✓ S ✓ R (2)

QUESTION 9

<p>9.1</p>	 <p>Construction: Connect DC and BE and draw the altitudes k and h</p> $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \times AD \times k}{\frac{1}{2} \times BD \times k} = \frac{AD}{BD}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC} = \frac{\frac{1}{2} \times AE \times h}{\frac{1}{2} \times EC \times h} = \frac{AE}{EC}$ <p>but/maar: Area $\triangle BDE$ = Area $\triangle DEC$ [DE common base and DE \parallel BC/ DE gemeensk basis en DE \parallel BC]</p> $\therefore \frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	<p>✓ constr ✓ S ✓ S ✓ S ✓ R ✓ conclusion</p> <p>(6)</p>
<p>9.2.1</p>	<p>tangent-chord theorem</p>	<p>✓ R (1)</p>
<p>9.2.2</p>	<p>In $\triangle ABC$ and $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $\therefore \triangle ABC \parallel \triangle ADB$ [\angle; \angle; \angle] OR In $\triangle ABC$ and $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $\hat{B}\hat{C}\hat{A} = \hat{B}_2$ [\angles of $\triangle = 180^\circ$] $\therefore \triangle ABC \parallel \triangle ADB$</p>	<p>✓ S ✓ S ✓ R ✓ S ✓ S ✓ R</p>
<p>9.2.3</p>	<p>$\hat{E}_2 = \hat{F}_1$ [verwiss \anglee/alternate \angles ; EA \parallel GF] $\hat{F}_1 = \hat{D}_2$ [ext \angle of cyc quad DGFC/buite \angle v kdvh DGFC] $\therefore \hat{E}_2 = \hat{D}_2$</p>	<p>✓ S ✓ R ✓ S ✓ R</p>

9.2.4	<p>In $\triangle AEC$ and $\triangle ADE$:</p> $\hat{A}_2 = \hat{A}_2 \quad [\text{common/gemeenskaplik}]$ $\hat{E}_2 = \hat{D}_2 \quad [\text{proven/bewys in 10.2.3}]$ $\therefore \triangle AEC \parallel \triangle ADE \quad [\angle ; \angle ; \angle]$ $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$ <p>OR</p> <p>In $\triangle AEC$ and $\triangle ADE$:</p> $\hat{A}_2 = \hat{A}_2 \quad [\text{common/gemeenskaplik}]$ $\hat{E}_2 = \hat{D}_2 \quad [\text{proven/bewys in 10.2.3}]$ $\hat{ACE} = \hat{G}_1 \quad [\angle \text{ of } \Delta = 180^\circ \text{ OR ext } \angle \text{ of cyc quad DGFC/}$ <p style="text-align: right;">buite \angle v kdvh DGFC]</p> $\therefore \triangle AEC \parallel \triangle ADE$ $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$	<p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>✓ S</p>
9.2.5	$\frac{AB}{AD} = \frac{AC}{AB} \quad [\triangle ABC \parallel \triangle ADB]$ $AB^2 = AD \times AC$ $= AE^2 \quad [\text{from 10.2.4}]$ $\therefore AB = AE$	<p>✓ S</p> <p>✓ S</p> <p>✓ S</p>
		[21]

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GRADE 12

MATHEMATICS P2

MARKING GUIDELINE

SEPTEMBER 2023

MARKS: 150

This marking guideline consists of 14 pages.

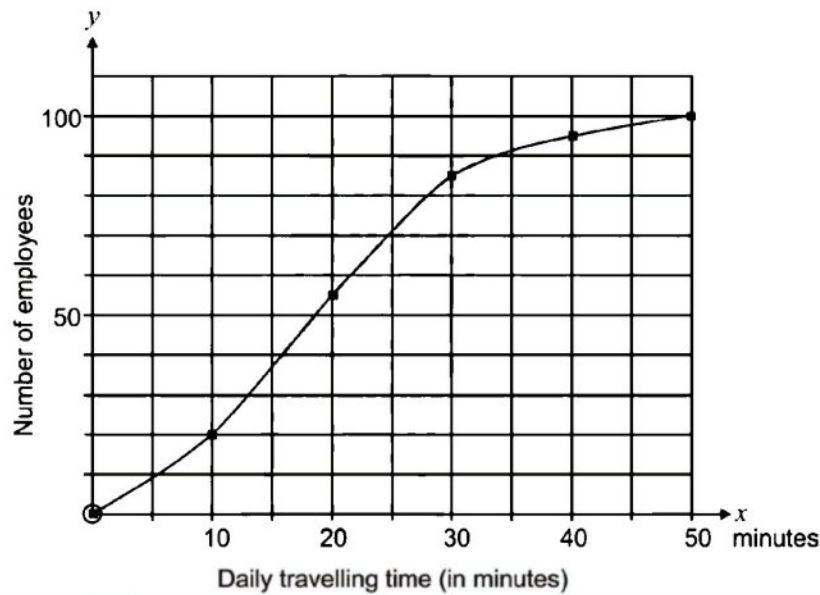
NB: CA APPLIES TO ALL SUB-QUESTIONS IN THIS MARKING GUIDELINE.

QUESTION 1

1.1	$a = 34,90$ $b = 0,08$ $y = 34,90 + 0,08x$ Answer only: Full Marks	✓A Value of a ✓A Value of b ✓CA Equation	(3)
1.2	$r = 0,88$	✓A✓A Answer	(2)
1.3	$y = 34,90 + 0,08x$ $y = 34,90 + 0,08(560)$ $y = 79,7$ minutes = 1.33 hours Total cost = R150 + R150 = R300	✓CA Substitution ✓CA 79,7 minutes ✓CA Answer	(3)
1.4.1	Area = $100 \times 70 = 7000$ square meters $y = 34,90 + 0,08(7000)$ $y = 594,9$ minutes = 9.92 hours	✓CA Calculation	(1)
1.4.2	No. The time taken will exceed his daily 8 hour working hours.	✓CA Justification	(1)
			[10]

QUESTION 2

Daily travelling time x (in minutes)	Number of employees (f)	Midpoint of Interval (x)	$f \cdot x$
$0 \leq x < 10$	20	5	100
$10 \leq x < 20$	35	15	525
$20 \leq x < 30$	30	25	750
$30 \leq x < 40$	10	35	350
$40 \leq x < 50$	5	45	225
Total	100		1950



2.1	Estimated Mean = $\frac{1950}{100} = 19,5$ Answer only: Full Marks	✓A 1950 ✓A 100 ✓CA Answer	(3)
2.2	$10 \leq x < 20$	✓A ✓A Answer	(2)
2.3	See Diagram	✓A Minimum and Maximum value ✓A 1 st and 3 rd Quartiles ✓A 2 nd Quartile	(3)
Five number summary: 0 ; 12 ; 18 ; 26 ; 50 Accept: ± 1 deviation on quartiles			

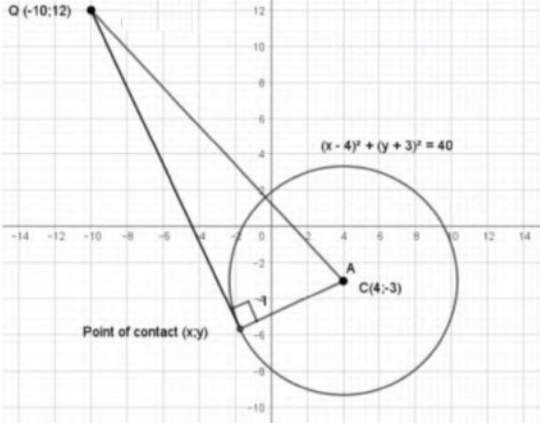
2.4.1	True	✓A	Answer	(1)
2.4.2	False	✓A	Answer	(1)
2.4.3	True	✓A	Answer	(1)
				[11]

QUESTION 3

3.1	$m_{AD} = \frac{7 + 3}{1 + 4} = \frac{10}{5} = 2$	✓A Substitution of points A and D ✓CA Answer	(2)
3.2	$m_{AD} = m_{BC} = 2 \dots\dots(AD \parallel BC)$ $y = mx + c$ $-8 = 2(-2) + c$ $-4 = c$ $y = 2x - 4$	✓CA Gradient of BC ✓CA Substitution of point C and gradient ✓CA Answer	(3)

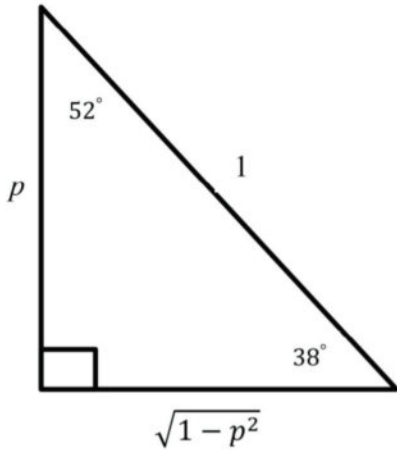
3.3	Let $y = 0$: $0 = 2x - 4$ $x = 2$ $F(2 ; 0)$	\checkmark CA $0 = 2x - 4$ \checkmark CA x - value	(2)
3.4	$M(3 ; 2)$	\checkmark A x - value \checkmark A y - value	(2)
3.5	$m_{CD} = \frac{-3 + 8}{-4 + 2} = \frac{5}{-2}$ Inclination of CD: $\tan \theta_1 = 180^\circ - 68,2^\circ = 111,8^\circ$ Inclination of CF: $\tan \theta_2 = 2$ $\theta_2 = 63,43^\circ$ Therefore $\alpha = 111,8^\circ - 63,43^\circ$ $\quad = 48,37^\circ$ <p style="text-align: center;">OR</p> $DF = \sqrt{(2 + 4)^2 + (0 + 3)^2}$ $\quad = \sqrt{45}$ $DC = \sqrt{29}$ $CF = \sqrt{80}$ $(\sqrt{45})^2 = (\sqrt{29})^2 + (\sqrt{80})^2 - 2\sqrt{29} \sqrt{80} \cos \alpha$ $\alpha = 48,37^\circ$	\checkmark A Gradient of CD \checkmark A Inclination of CD \checkmark A Inclination of CF \checkmark A subtraction of the angles \checkmark A Distance of DF \checkmark A Distance of DC \checkmark A Distance of CF \checkmark A subst. into cosine rule	(4)
3.6	$DC = \sqrt{(-3 + 8)^2 + (-4 + 2)^2} = \sqrt{29}$ $FC = \sqrt{(0 + 8)^2 + (2 + 2)^2} = \sqrt{80}$ $\text{Area of } \triangle DCF = \frac{1}{2}(\sqrt{29})(\sqrt{80}) \sin 48,37^\circ$ $\quad = 18 \text{ square units.}$	\checkmark A Length of CD \checkmark CA Length of FC \checkmark CA Substitution into area formula \checkmark CA Answer	(4)
			[17]

QUESTION 4

<p>4.1.1</p>	$x^2 + y^2 - 8x + 6y = 15$ $\text{LHS} = (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ $= \text{RHS}$	<p>✓A Subst. of point ✓A Simplification</p>	<p>(2)</p>
<p>4.1.2</p>	 <p>$(x - 4)^2 + (y + 3)^2 = 40$</p> <p>Centre: C(4 ; -3) P(2 ; -9)</p> $m_{\text{Radius}} = \frac{-3 + 9}{4 - 2} = \frac{6}{2} = 3$ $m_{\text{Tangent}} = -\frac{1}{3}$ <p>Equation of Tangent:</p> $y = mx + c$ $-9 = -\frac{1}{3}(2) + c$ $-\frac{25}{3} = c$ $y = -\frac{1}{3}x - \frac{25}{3}$	<p>✓A writing the equation as $(x - 4)^2 + (y + 3)^2 = 40$ ✓CA Centre of circle ✓CA Gradient of radius ✓CA Gradient of tangent</p> <p>✓CA Substitution</p> <p>✓CA Answer</p>	<p>(6)</p>
<p>4.1.3</p>	$r^2: (x - 4)^2 + (y + 3)^2 = 40$ $r^2 = 40$ $(\text{distance } Q \text{ to the centre})^2 = (-10 - 4)^2 + (12 + 3)^2$ $= 421$ $(\text{Length of tangent})^2 = 421 - 40 = 381$ $\text{Length of tangent} = \sqrt{381}$	<p>✓CA Calculation of r^2</p> <p>✓CA distance calculation ✓CA Tangent calculation</p> <p>✓CA Answer</p>	<p>(4)</p>

4.2.1	$(x - 3)^2 + (y + 2)^2 = 25$ Let $x = 0$: $(0 - 3)^2 + (y + 2)^2 = 25$ $(y + 2)^2 = 16$ $y + 2 = \pm 4$ $y = -6$ or $y = 2$ $B(0; 2)$	✓A Letting $x = 0$ ✓A Simplification ✓CA y - values ✓CA Answer	(4)
4.2.2	$C(6; 2)$	✓CA x - value ✓CA y - value	(2)
4.2.3 (a)	$T(3; -2)$ and $M(12; 10)$ $TM^2 = (12 - 3)^2 + (10 + 2)^2 = 225$ $TM = 15$ units	✓A Coordinates of M ✓CA Answer	(2)
(b)	Radius, center T = 5 units and Radius, center M = 10 units Sum of radii = 15 units Circles touch . $TM =$ Sum of radii	✓CA Sum of radii ✓CA Justification	(2)
			[22]

QUESTION 5

5.1			
5.1.1	$\cos 218^\circ$ $= -\cos 38^\circ$ $= -\frac{\sqrt{1-p^2}}{1}$	✓A Calculation of $\sqrt{1-p^2}$ ✓A Reduction ✓CA Answer	(3)

5.1.2	$\begin{aligned} \cos 14^\circ &= \cos(52^\circ - 38^\circ) \\ &= \cos 52^\circ \cos 38^\circ + \sin 52^\circ \sin 38^\circ \\ &= \left(\frac{p}{1}\right) \left(\frac{\sqrt{1-p^2}}{1}\right) + \left(\frac{\sqrt{1-p^2}}{1}\right) \left(\frac{p}{1}\right) \\ &= 2p\sqrt{1-p^2} \end{aligned}$ <p style="text-align: center;">OR</p> $\begin{aligned} \cos 24^\circ &= \sin 76^\circ \\ &= \sin 2(38^\circ) \\ &= 2 \sin 38^\circ \cos 38^\circ \\ &= 2p\sqrt{1-p^2} \end{aligned}$	<p>✓A Writing as difference</p> <p>✓A Expansion</p> <p>✓CA Answer</p> <p>✓ sin 76°</p> <p>✓ double angle</p> <p>✓ answer</p>	(3)
5.1.3	$\begin{aligned} \sin 26^\circ \cos 26^\circ &= \frac{1}{2} \sin 52^\circ \\ &= \frac{1}{2} \sqrt{1-p^2} \end{aligned}$	<p>✓A Double angle</p> <p>✓CA Answer</p>	(2)
5.2	$\begin{aligned} &\frac{2 \sin 165^\circ \cos 195^\circ}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ} \\ &= \frac{2 \sin 15^\circ \cdot (-\cos 15^\circ)}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ} \\ &= \frac{-2 \sin 30^\circ}{\sin(15^\circ - 45^\circ)} \\ &= \frac{-2 \sin 30^\circ}{\sin(-30^\circ)} \\ &= \frac{-2 \sin 30^\circ}{-\sin 30^\circ} \\ &= 2 \end{aligned}$	<p>✓A $-\cos 15^\circ$</p> <p>✓A $-2 \sin 30^\circ$</p> <p>✓A $\sin(15^\circ - 45^\circ)$</p> <p>✓A $-\sin 30^\circ$</p> <p>✓CA Answer</p>	(5)
5.3.1	$\begin{aligned} K &= \sqrt{3} \cos x + \sin x \\ K &= 2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right) \\ K &= 2(\sin 60^\circ \cos x + \cos 60^\circ \sin x) \\ K &= 2 \sin(60^\circ + x) \end{aligned}$	<p>✓A $2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right)$</p> <p>✓A $\sin 60^\circ$ and $\cos 60^\circ$</p> <p>✓A $2 \sin(60^\circ + x)$</p>	(3)

5.3.2	$t = 2$ and $\theta = 60^\circ$	✓CA t -value and $\theta = 60^\circ$	(1)
5.3.3	2	✓CA Answer	(1)
5.4.1	$\text{LHS} = \frac{2 \tan \theta - \sin 2\theta}{2 \sin^2 \theta}$ $= \frac{2 \left(\frac{\sin \theta}{\cos \theta} \right) - 2 \sin \theta \cos \theta}{2 \sin^2 \theta} \times \frac{\cos \theta}{\cos \theta}$ $= \frac{2 \sin \theta - 2 \sin \theta \cos^2 \theta}{2 \sin^2 \theta \cos \theta}$ $= \frac{2 \sin \theta (1 - \cos^2 \theta)}{2 \sin^2 \theta \cos \theta}$ $= \frac{2 \sin \theta \cdot \sin^2 \theta}{2 \sin^2 \theta \cdot \cos \theta}$ $= \frac{\sin \theta}{\cos \theta}$ $= \tan \theta$ $= \text{LHS}$	✓A $\frac{\sin \theta}{\cos \theta}$ ✓A $2 \sin \theta \cos \theta$ ✓A simplification ✓A factorizing ✓A $1 - \cos^2 \theta = \sin^2 \theta$ ✓A simplified to $\frac{\sin \theta}{\cos \theta}$	(6)
5.4.2	$2 \sin^2 \theta = 0$ $\sin \theta = 0$ $\therefore \theta = 180^\circ$ and 360° $\theta = 270^\circ$	✓ 180° and 360° ✓ 270°	(2)
			[26]

QUESTION 6

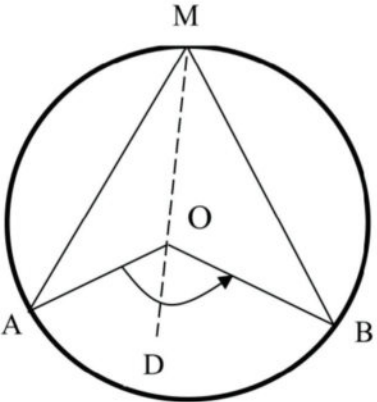
6.1			
<p>Graph of f: 1A mark for x – intercepts 1A marks for minimum and maximum points 1A mark for shape</p> <p>Graph of g: 1A mark for end points 1A mark for x – intercepts 1A mark for y – intercept</p> <p style="text-align: right;">(6)</p>			
6.2.1	360°	✓A Answer	(1)
6.2.2	$x \in [-180^\circ; -150^\circ) \cup (30^\circ; 180^\circ]$	$[-180^\circ; -150^\circ)$ ✓A : Notation ✓CA : values $(30^\circ; 180^\circ]$ ✓A : Notation ✓CA : values	(4)
6.2.3	$f(x) = 1.5 + g(x)$ $f(x) - g(x) = 1.5$ $x = 90^\circ$ or $x = 150^\circ$	✓A $f(x) - g(x) = 1.5$ ✓A Answer	(2)
			[13]

QUESTION 7

7.1	$\frac{PQ}{QR} = \tan \theta$ $PQ = QR \tan \theta$	✓ A Answer	(1)
7.2	$\frac{QR}{\sin \hat{S}} = \frac{QS}{\sin \hat{R}}$ $\frac{QR}{\sin \beta} = \frac{x}{\sin \alpha}$ $QR = \frac{x \sin \beta}{\sin \alpha}$ $PQ = \frac{x \sin \beta \tan \theta}{\sin \alpha}$	✓ A Sine rule formula ✓ A Subs. Sine rule ✓ A Making QR a subj. of the formula ✓ A Subst. of QR	(4)
7.3	$\text{Area of } \triangle QSR = \frac{1}{2}(x)(8 - 2x) \sin 60^\circ$ $= \frac{1}{2}(x)(8 - 2x) \left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}(x)(8 - 2x)$ $= \sqrt{3}(x) \left(2 - \frac{1}{2}x\right)$ $= 2\sqrt{3}x - \frac{\sqrt{3}}{2}x^2$	✓ A Subst. into Area rule ✓ A $\frac{\sqrt{3}}{2}$ ✓ A Simplifying	(3)

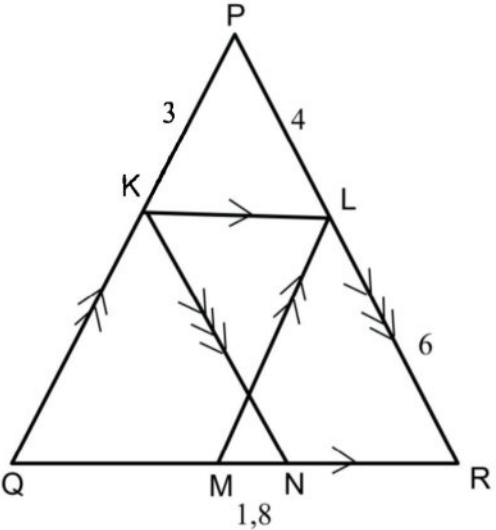
7.4	<p>For Max Area: $x = -\frac{b}{2a}$</p> $x = -\frac{(2\sqrt{3})}{2\left(-\frac{\sqrt{3}}{2}\right)}$ $x = 2$	<p>✓ A Formula</p> <p>✓ A Substitution into formula</p> <p>✓ CA Answer</p>	(3)
			[11]

QUESTION 8

8.1	 <p><u>Constr:</u> Join MO and produce to D.</p> <p>$\widehat{AOD} = \widehat{OAM} + \widehat{AMO} \dots$ (Ext. \angle of Δ)</p> <p>$\widehat{BOD} = \widehat{OBM} + \widehat{BMO} \dots$ (Ext. \angle of Δ)</p> <p>But $\widehat{OAM} = \widehat{AMO}$ and $\widehat{OBM} = \widehat{BMO} \dots$ (Radii =)</p> <p>$\therefore \widehat{AOD} + \widehat{BOD} = 2\widehat{AMO} + 2\widehat{BMO}$</p> <p>$\widehat{AOB} = 2(\widehat{AMO} + \widehat{BMO})$</p> <p>$\widehat{AOB} = 2\widehat{M}$</p> <p>NOTE No construction : No marks</p>	<p>✓ A Construction</p> <p>✓ A S/R</p> <p>✓ A S</p> <p>✓ A S</p> <p>✓ A S</p>	(5)
8.2.1	<p>$\widehat{OVA} = \widehat{ODA} = 90^\circ \dots$ (Radius \perp Tangent)</p> <p>VODA is a cyclic quad. (Converse of opposite angles of quad. Supplementary)</p>	<p>✓ A S/R</p> <p>✓ A R</p>	(2)
8.2.2	<p>$\widehat{O}_1 = 40^\circ \dots$ (Exterior angle of cyclic quad = int. opp. Angle)</p>	<p>✓ S ✓ R</p>	(2)

<p>8.2.3</p>	<p>$\widehat{V}_1 = \frac{180^\circ - 40^\circ}{2} = 70^\circ$(sum of $\angle s$ of Δ; radii)</p> <p>$\widehat{A\widehat{D}M} = \frac{180^\circ - 40^\circ}{2} = 70^\circ$(sum of $\angle s$ of Δ; Tangents drawn from a common point A)</p> <p>$\widehat{A\widehat{D}M} = \widehat{O}_2 = 70^\circ$... (Angles subtended by common chord AV)</p> <p>BV OA(Converse of Alt $\angle s$ or Alt $\angle s$ are =)</p> <p style="text-align: center;">OR</p> <p>In ΔOVA and ΔODA OV=OD.....(radii) OA=OA.....(common) AV=AD.....(tangents from the same point) $\Delta OVA \equiv \Delta ODA$.....(SSS) $A_1 = A_2$.....($\equiv \Delta s$) $= 20^\circ$ $A_2 = V_2$.....($\angle s$ in the same segment) $= 20^\circ$ $V_1 = 90^\circ - 20^\circ = 70^\circ$ $O_2 = 180^\circ - [OVA + \widehat{A_1}]$ $= 180^\circ - [90^\circ + 20^\circ]$ $= 70^\circ$ $\therefore \widehat{V_1} = \widehat{O_2}$.....(both = 70°) $\therefore BV \parallel OA$..... (alt $\angle s$ are = or conv.alt $\angle s$)</p>	<p>✓ S/R</p> <p>✓ S/R</p> <p>✓S ✓R</p> <p>✓ R</p> <p>✓A $\Delta OVA \equiv \Delta ODA$.....(SSS)</p> <p>✓A S/R</p> <p>✓A $V_1 = 70^\circ$</p> <p>✓A $O_2 = 70^\circ$</p> <p>✓A R</p>	<p>(5)</p>
			<p>[14]</p>

QUESTION 9

9.1	Divides the other two sides, proportionally.	✓S <u>divides</u> the other two sides ✓S <u>proportionally</u>	(2)
9.2			

9.2.1	$\frac{KQ}{3} = \frac{6}{4} \dots\dots\dots(\text{Prop. Thm; } KL \parallel QR)$ KQ = 4,5 units	✓ S/R ✓ Answer	(2)
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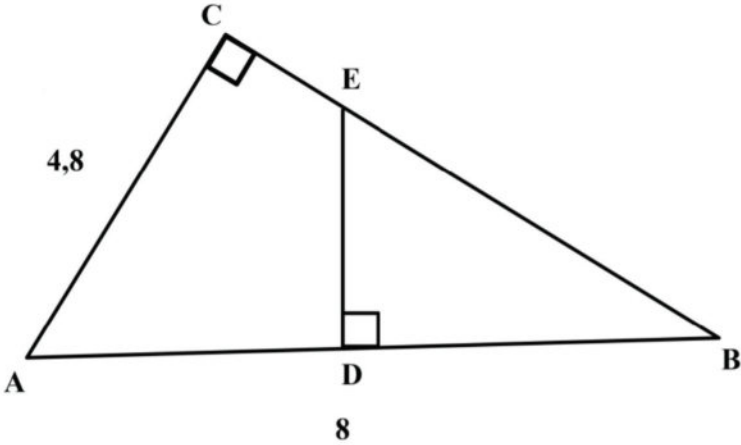
9.2.2	KL = QM(Opposite sides of \parallel^m QKLM are equal) KL = NR (Opposite sides of \parallel^m NKLR are equal) Therefore QM = NR (both = KL) <p style="text-align: center;">OR</p> $\frac{QN}{QR} = \frac{QK}{KP} \quad \text{Prop Thm, } KN \parallel PR$ $\frac{4,5}{3} = \frac{3}{2}$ $\frac{MR}{QM} = \frac{RL}{PL} \quad \text{Prop Thm, } ML \parallel QP$ $= \frac{6}{4} = \frac{3}{2}$ $\therefore \frac{QN}{NR} = \frac{MR}{QM}$ $\frac{QM+1,8}{NR} = \frac{NR+1,8}{QM}$ $\therefore QM(QM+1,8) = MR(NR+1,8)$ $\therefore QM = NR$	✓ S/R ✓ S/R $\checkmark \frac{QN}{QR} = \frac{QK}{KP} \quad \text{Prop Thm, } KN \parallel PR$ <p style="text-align: center;">OR</p> $\frac{MR}{QM} = \frac{RL}{PL} \quad \text{Prop Thm, } ML \parallel QP$ $\checkmark \frac{QM+1,8}{NR} = \frac{NR+1,8}{QM}$	(2)
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[16]

QUESTION 10

<p>10.1</p>	<p>In Δ's MQA and MBQ $\widehat{M}_1 = \widehat{M}_1$(Common) $\widehat{A}_1 = \widehat{Q}_1$(Tan-Chord Theorem) $\widehat{AQM} = \widehat{B}_1$(Remaining \angles of Δ's) $\Delta MQA \parallel \Delta MBQ$(AAA)</p>	<p>✓ S/R ✓ S/R ✓ R</p>	<p>(3)</p>
<p>10.2</p>	<p>In Δ's MAR and MRB $\widehat{M}_2 = \widehat{M}_2$(Common) $\widehat{A}_2 = \widehat{R}_1$(Tan-Chord Thm) $\widehat{ARM} = \widehat{RBM}$(Remaining \angles of Δ's) $\Delta MAR \parallel \Delta MRB$(AAA) $\frac{MA}{MR} = \frac{MR}{MB}$(Δ's similar) $MR^2 = AM \cdot MB$</p>	<p>✓ S ✓ S/R ✓ R ✓ S A✓R</p>	<p>(5)</p>
<p>10.3</p>	<p>$\frac{MQ}{MB} = \frac{MA}{MQ}$(from 10.1) $MQ^2 = MB \cdot MA = \dots \dots (1)$ Also $MR^2 = AM \cdot MB$ Now $\frac{MQ^2}{MR^2} = \frac{MB \cdot MA}{AM \cdot MB}$ $\therefore \frac{MQ^2}{MR^2} = 1$</p>	<p>✓ $\frac{MQ}{MB} = \frac{MA}{MQ}$ ✓ $MQ^2 = MB \cdot MA$ ✓✓ $\frac{MQ^2}{MR^2} = \frac{MB \cdot MA}{AM \cdot MB}$</p>	<p>(4)</p>
			<p>[12]</p>

QUESTION 11

			
11.1	$BC^2 = 8^2 - 4,8^2 \dots\dots\dots(\text{TOP})$ $BC = 6,4 \text{ cm}$	✓ S ✓ S	(2)
11.2	ΔBED	✓ S	(1)
11.3	$\frac{BA}{BE} = \frac{AC}{ED} = \frac{BC}{BD}$ $\frac{8}{BE} = \frac{4,8}{ED} = \frac{6,4}{4}$ $ED = \frac{4,8 \times 4}{6,4} = 3 \text{ cm}$ Area of ΔEDB $= \frac{1}{2} (4)(3) = 6 \text{ cm}^2$ Area of ΔABC $= \frac{1}{2} (4,8)(6,4) = 15,6 \text{ cm}^2$ Therefore, Area of ADEC $= 15,6 - 6 = 9,6 \text{ cm}^2$	$\Delta BAC \parallel \Delta BED$ ✓A S ✓CA Value of ED ✓CA Area of ΔEDB ✓CA Area of ΔABC ✓CA Area of ADEC	(5)
		[8]	

GRAND TOTAL: 150

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GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2023

MARKING GUIDELINES

MARKS: 150

NOTE:

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CAPS/KABV – Grade/Graad 12 – Marking Guideline/Nasienriglyn

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

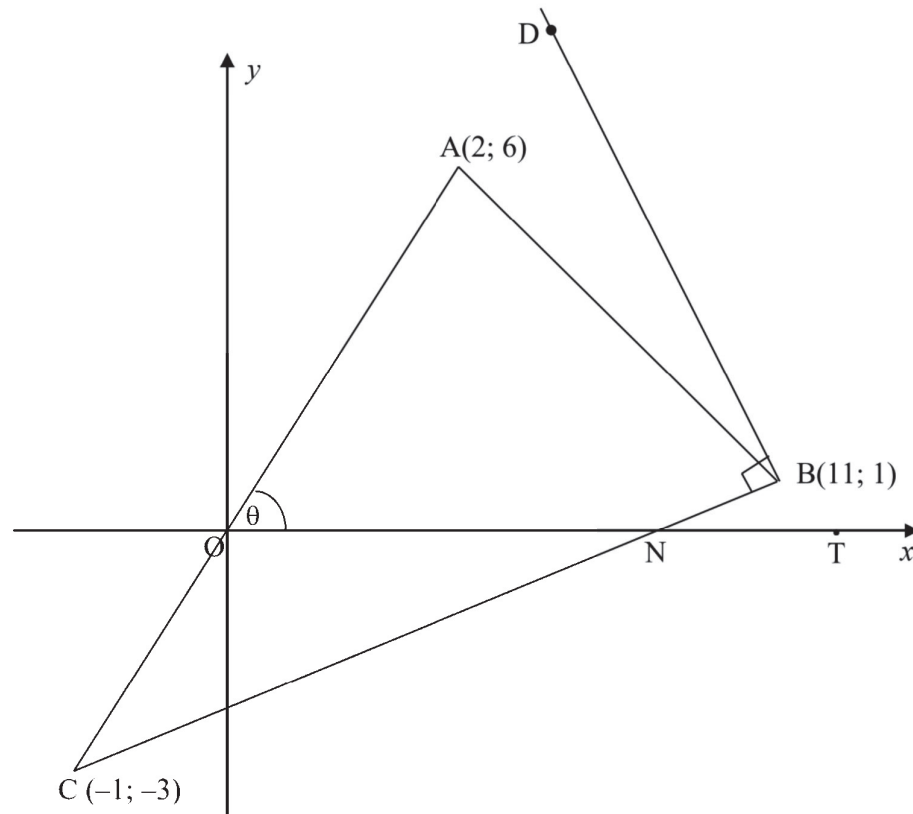
QUESTION/VRAAG 1

71	83	88	91	92	92	95	97	104	108	109	110	111	115	129
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1.1	$\frac{1495}{15} = 99,67$	✓✓ 99,67 (2)
1.2	$\sigma = 14,06$	✓SD (1)
1.3	$99,67 - 14,06 = 85,61$ $\frac{2}{15} \times 100 = 13,33\%$	✓boundary ✓ $\frac{2}{15}$ ✓answer (3)
1.4.1	96,67	✓answer (1)
1.4.2	14,06	✓answer (1)
		[8]

QUESTION/VRAAG 2

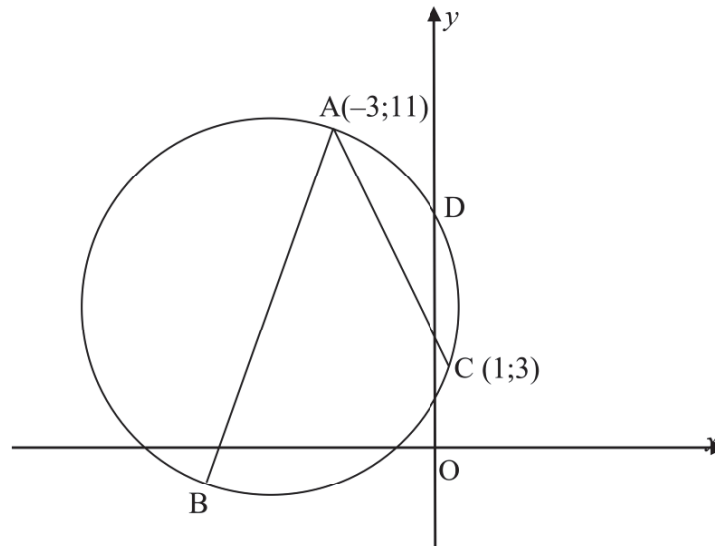
2.1.1	True	✓ answer (1)
2.1.2	Positively skewed / Skewed to the right	✓ answer (1)
2.1.3	Range for company A = $800 - 200 = 600$ Range for company B = $600 - 100 = 500$ Biggest range : Company A	✓ both ranges ✓ Company A (2)
2.1.4	$20 \times 75\% = 15$ workers	✓ 75% ✓ 15 workers (2)
2.2.1	$y = 5965,51 - 2,93x$	✓ 5329,84 ✓ -2,61 ✓ equation (3)
2.2.2	$r = -0,49$	✓ (1)
2.2.3	$y = 5965,51 - 2,93(2018)$ $= 52,77$ The correlation coefficient is moderate thus the value is fairly reliably predicted.	✓ prediction ✓ answer + reason (2)
		[12]

QUESTION/VRAAG 3

3.1.1	<p>$N : (x; 0)$</p> $m_{NC} = \frac{0 - (-3)}{x - (-1)} = \frac{1}{3}$ $x + 1 = 9$ $x = 8$ <p>OR</p> $y - (-3) = \frac{1}{3}(x - (-1))$ $y = \frac{1}{3}x - \frac{8}{3}$ <p>For $y = 0 : x = 8$</p>	<p>✓ subst into gradient formula</p> <p>✓ answer (2)</p> <p>✓ subst into equation</p> <p>✓ answer</p>
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CAPS/KABV – Grade/Graad 12 – Marking Guideline/Nasiemriglyn

3.1.2	$\tan \theta = 3$ $\theta = 71,565\dots^\circ$ $\tan \hat{BNT} = \frac{1}{3}$ $\hat{BNT} = 18,434\dots^\circ$ $\hat{C} = 71,565\dots^\circ - 18,434\dots^\circ$ $= 53,13^\circ$ Accept $\hat{C} = 53,14^\circ$	✓ substitution ✓ θ ✓ substitution ✓ \hat{BNT} ✓ answer (5)
3.2	$y - 6 = 3(x - 2)$ $y = 3x$	✓ substitution ✓ equation (2)
3.3	$m_{\perp} = -3$ $y - 1 = -3(x - 11)$ $y = -3x + 34$ $3x = -3x + 34$ $6x = 34$ $x = \frac{17}{3} = 5,67$ $y = 3\left(\frac{17}{3}\right) = 17$	✓ $m_{\perp} = -3$ ✓ equation ✓ setting equations equal ✓ value of x ✓ value of y (5)
		[14]

QUESTION/VRAAG 4

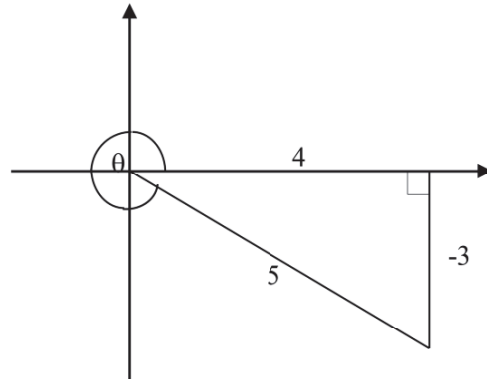
4.1	$m_{AC} = \frac{11 - 3}{-3 - 1} = \frac{8}{-4} = -2$ $mdpt_{AC} = (-1; 7)$ $y - 7 = \frac{1}{2}(x - (-1))$ $y = \frac{1}{2}x + \frac{15}{2}$	✓ gradient of AC. ✓ Midpt AC ✓ ⊥ gradient and subst ✓ equation (4)
4.2	$\frac{1}{2}x + \frac{15}{2} = 3x + 20$ $- 2,5x = 12,5$ $x = -5$ $y = 3(-5) + 20 = 5$ $\therefore \text{centre}(-5; 5)$	✓ set equations equal ✓ solve x ✓ solve y (3)
4.3	$\text{radius} = \sqrt{(-5 - (-3))^2 + (5 - 11)^2}$ $= \sqrt{40} = 2\sqrt{10}$ $\text{diameter} = 4\sqrt{10} \text{ units}$ $= 12,65$	✓ substitution in formula ✓ radius ✓ diameter (3)
4.4	$(x + 5)^2 + (y - 5)^2 = 40$	✓ $(x + 5)$, $(y - 5)$ ✓ 40 (2)

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4.5	$m_{rad} = \frac{11-5}{-3-(-5)} = 3$ $m_{tan} = -\frac{1}{3}$ $y - 11 = -\frac{1}{3}(x - (-3))$ $y = -\frac{1}{3}x + 10$ $(0; p): p = -\frac{1}{3}(0) + 10$ $p = 10$	<p>✓ gradient</p> <p>✓ subst in form</p> <p>✓ equation</p> <p>✓ value of p</p> <p>(4)</p>
4.6	$(x + 2)^2 + (y - 7)^2 = (\sqrt{10})^2$ $(x + 2)^2 + (y - 7)^2 = 10$	<p>✓(x + 2)</p> <p>✓(y - 7)</p> <p>✓10</p> <p>(3)</p>
4.7	<p>Centres: (2; 3) and (-5; 5)</p> <p>Distance between centres = $\sqrt{(2 - (-5))^2 + (3 - 5)^2}$</p> <p style="padding-left: 40px;">$= \sqrt{53} = 7,28$</p> <p>Sum of radii = $2 + 2\sqrt{10} = 8,32$</p> <p>Distance between centres < Sum of radii</p> <p>∴ They will intersect.</p>	<p>✓ subst of centres</p> <p>✓ distance</p> <p>✓ Sum of radii</p> <p>✓ conclusion</p> <p>(4)</p>
		[23]

QUESTION 5

5.1



5.1.1	$(-3)^2 + 4^2 = r^2$ $r^2 = 25$ $r = 5$ $\sin \theta = \frac{-3}{5}$	✓ value of r ✓ answer (2)
5.1.2	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= \left(\frac{4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$ $= \frac{16}{25} - \frac{9}{25}$ $= \frac{7}{25}$	✓ expansion ✓ substitution ✓ answer (3)
5.1.3	$\cos(\theta + 30^\circ) = \cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ$ $= \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{-3}{5}\right)\left(\frac{1}{2}\right)$ $= \frac{4\sqrt{3} + 3}{10}$	✓ expansion ✓ substitution ✓ answer (3)
5.2	$(4 \sin \alpha)^2 + (4 \cos \alpha)^2$ $= 16 \sin^2 \alpha + 16 \cos^2 \alpha$ $= 16(\sin^2 \alpha + \cos^2 \alpha)$ $= 16(1) = 16$	✓ simplification ✓ answer (2)

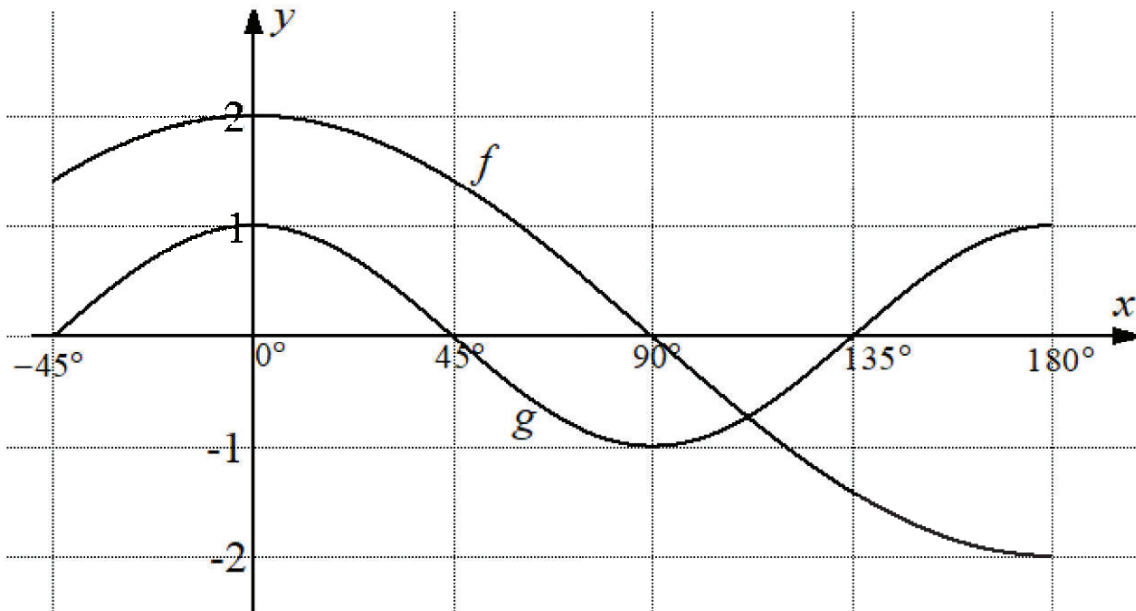
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5.3	$\begin{aligned} & \sin(90^\circ - x) \cdot \cos(-x) - \sin(x - 180^\circ) \cdot \sin(90^\circ + x) \\ &= \sin(180^\circ - x) \cdot \cos(360^\circ - x) - \sin(180^\circ + x) \cdot \sin(90^\circ + x) \\ &= \sin x \cos x - (-\sin x) \cos x \\ &= \sin x \cos x + \sin x \cos x \\ &= 2 \sin x \cos x \\ &= \sin 2x \end{aligned}$	$\checkmark \sin x$ $\checkmark \cos x$ $\checkmark -\sin x$ $\checkmark \cos x$ \checkmark simplification \checkmark double angle identity (6)
5.4.1	$\frac{\sin 7x + \sin x}{2 \cos 3x} = \sin 4x$ $\text{LHS} = \frac{\sin(4x + 3x) + \sin(4x - 3x)}{2 \cos 3x}$ $= \frac{\sin 4x \cos 3x + \cos 4x \sin 3x + \sin 4x \cos 3x - \cos 4x \sin 3x}{2 \cos 3x}$ $= \frac{\sin 4x \cos 3x + \sin 4x \cos 3x}{2 \cos 3x}$ $= \frac{2 \sin 4x \cos 3x}{2 \cos 3x}$ $= \sin 4x = \text{RHS}$	\checkmark both brackets $\checkmark \sin(4x + 3x)$ expansion $\checkmark \sin(4x - 3x)$ expansion \checkmark simplification (4)
5.4.2	$2 \cos 3x = 0$ $\cos 3x = 0$ $3x = \begin{cases} 90^\circ + 360^\circ k \\ -90^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z}$ $x = \begin{cases} 30^\circ + 120^\circ k \\ -30^\circ + 120^\circ k \end{cases}$ <p style="text-align: center;">OR</p> $2 \cos 3x = 0$ $\cos 3x = 0$ $3x = \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z}$ $x = \begin{cases} 30^\circ + 120^\circ k \\ 90^\circ + 120^\circ k \end{cases}$	$\checkmark 2 \cos 3x = 0$ $\checkmark 3x = \begin{cases} 90^\circ + 360^\circ k \\ -90^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z}$ $\checkmark x = \begin{cases} 30^\circ + 120^\circ k \\ -30^\circ + 120^\circ k \end{cases}$ <p style="text-align: center;">OR</p> $\checkmark 2 \cos 3x = 0$ $\checkmark 3x = \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z}$ $\checkmark x = \begin{cases} 30^\circ + 120^\circ k \\ 90^\circ + 120^\circ k \end{cases}$ (3)

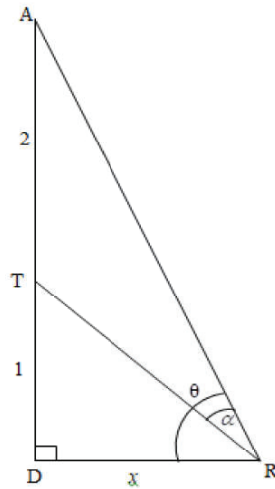
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5.5	$\sin(3x + 20^\circ) = \cos x$ $\sin(3x + 20^\circ) = \sin(90^\circ - x)$ $3x + 20^\circ = 90^\circ - x + 360^\circ k \quad k \in \mathbb{Z}$ $4x = 70^\circ + 360^\circ k$ $x = 17,5^\circ + 90^\circ k$ <p style="text-align: center;"><i>OR</i></p> $3x + 20^\circ = 180^\circ - (90^\circ - x) + 360^\circ k \quad k \in \mathbb{Z}$ $3x + 20^\circ = 180^\circ - 90^\circ + x + 360^\circ k$ $2x = 70^\circ + 360^\circ k$ $x = 35^\circ + 180^\circ k$ <p style="text-align: center;"><i>OR</i></p> $\sin(3x + 20^\circ) = \cos x$ $\cos[90^\circ - (3x + 20^\circ)] = \cos x$ $-3x + 70^\circ = x + 360^\circ k \quad k \in \mathbb{Z}$ $-4x = -70^\circ + 360^\circ k$ $x = 17,5^\circ + 90^\circ k$ <p style="text-align: center;"><i>OR</i></p> $-3x + 70^\circ = -x + 360^\circ k \quad k \in \mathbb{Z}$ $-2x = -70^\circ + 360^\circ k$ $x = 35^\circ + 180^\circ k$	$\checkmark \sin(90 - x)$ $\checkmark 3x + 20^\circ = 90^\circ - x + 360^\circ k \quad k \in \mathbb{Z}$ $\checkmark x = 17,5^\circ + 90^\circ k$ $\checkmark 3x + 20^\circ = 180^\circ - (90^\circ - x) + 360^\circ k$ $\checkmark 2x = 70^\circ + 360^\circ k$ $\checkmark x = 35^\circ + 180^\circ k$ <p style="text-align: right;">(6)</p> $\checkmark \cos[90^\circ - (3x + 20^\circ)]$ $\checkmark -4x = -70^\circ + 360^\circ k$ $\checkmark x = 17,5^\circ + 90^\circ k$ $\checkmark -3x + 70^\circ = -x + 360^\circ k \quad k \in \mathbb{Z}$ $\checkmark -2x = -70^\circ + 360^\circ k$ $\checkmark x = 35^\circ + 180^\circ k$
		[29]

QUESTION 6



6.1	180°	✓ answer (1)
6.2		✓ critical values ✓ notation (2)
6.3	Range of g : $y \in [-1; 1]$ Range of $3g$: $y \in [-3; 3]$ Range of $y = 3g(x) - 1$: $-4 \leq y \leq 2$ or $y \in [-4; 2]$	✓ critical values ✓ notation (2)
6.4	$2 \cos x = \frac{1}{2}$ $\cos x = \frac{1}{4}$ $x = 75,5^\circ$ $-45^\circ \leq x \leq 75,5^\circ$	✓ equation ✓ $x = 75,5^\circ$ ✓ critical values ✓ notation (4)
6.5	$\frac{1}{2} \cos^2 x - \frac{1}{4}$ $= \frac{1}{4} (2 \cos^2 x - 1)$ $= \frac{1}{4} (\cos 2x)$ $= \frac{1}{4} (-1)$ $= -\frac{1}{4}$	✓ factor ✓ double angle ✓ min of $\cos 2x$ ✓ answer (4)

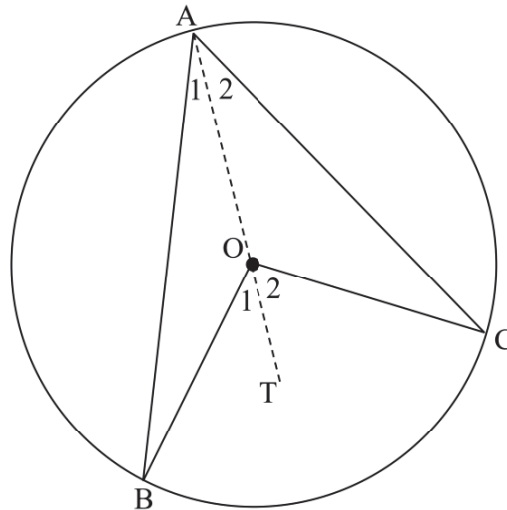
[13]**QUESTION 7**

7.1	$\hat{TRD} = \theta - \alpha$ $\cos(\theta - \alpha) = \frac{x}{TR}$ $TR = \frac{x}{\cos(\theta - \alpha)}$	$\checkmark \hat{TRD}$ $\checkmark \cos \text{ definition}$ (2)
7.2	$\hat{A} = 90^\circ - \theta$ $\frac{TR}{\sin(90^\circ - \theta)} = \frac{AT}{\sin \alpha}$ $\frac{TR}{\cos \theta} = \frac{2}{\sin \alpha}$ $TR = \frac{2 \cos \theta}{\sin \alpha}$ $\frac{x}{\cos(\theta - \alpha)} = \frac{2 \cos \theta}{\sin \alpha}$ $x = \frac{2 \cos \theta \cos(\theta - \alpha)}{\sin \alpha}$	$\checkmark x$ $\checkmark TR$ $\checkmark \text{ Subst in area-rule}$ $\checkmark \text{ answer}$ (4)

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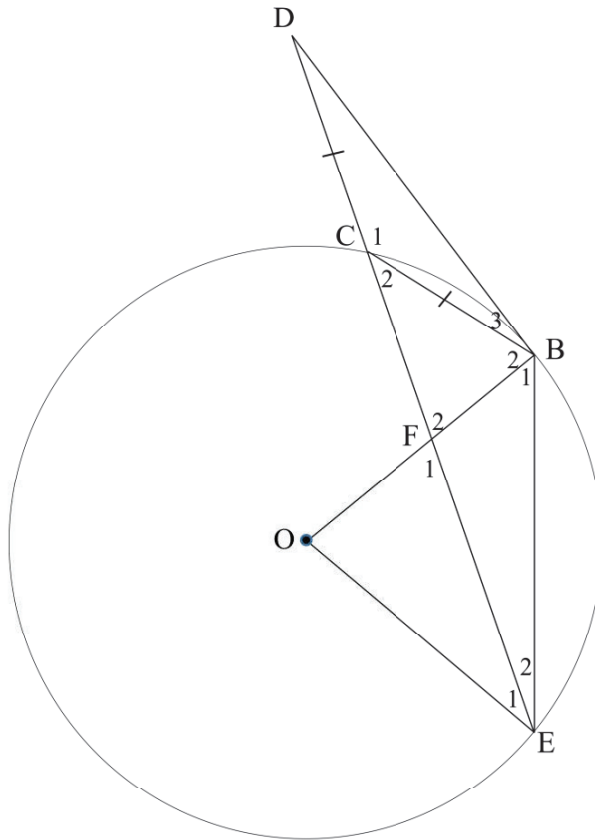
7.3	$x = \frac{2 \cos 68,33 \cos(68,33 - 28^\circ)}{\sin 28^\circ}$ $= 1,1992\dots$ $TR = \frac{1,1992}{\cos(\theta - \alpha)}$ $= 1,5730\dots$ $\hat{A} = 21,67^\circ$ $\hat{ATR} = 130,33^\circ$ $\text{Area } \Delta ATR = \frac{1}{2}(2)(1,5730)\sin 130,33^\circ$ $= 1,1992\dots$ $= 1,20 \text{ units}^2$	<p>✓ x</p> <p>✓ TR</p> <p>✓ subst in area formula</p> <p>✓ answer (4)</p>
		[10]

QUESTION 8



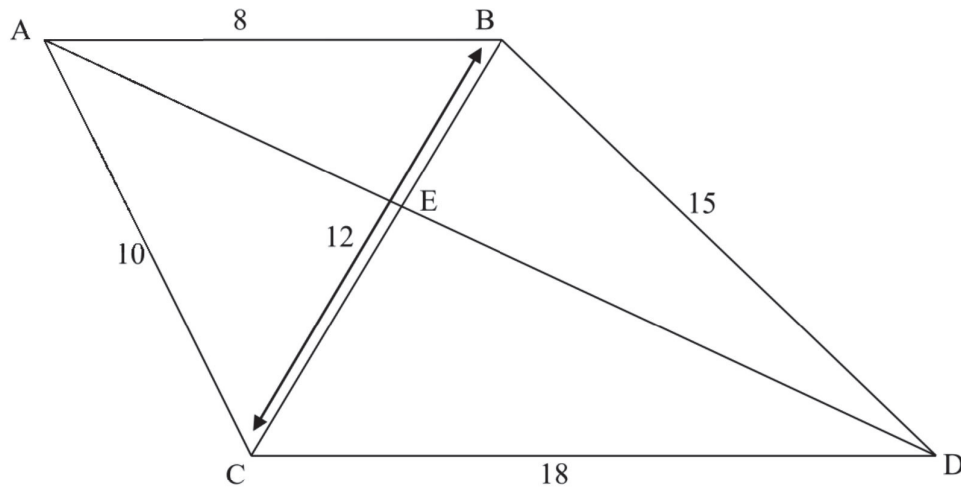
8.1	<p>Draw line from A through O to T</p> <p>let $\hat{A}_1 = x$ and $\hat{A}_2 = y$</p> <p>$\hat{A}_1 = \hat{B} = x$ (\angle's opp = sides)</p> <p>$\hat{O}_1 = 2x$ (<i>ext</i> \angle of Δ)</p> <p>similarly $\hat{O}_2 = 2y$</p> <p>$\hat{BOC} = 2x + 2y = 2(x + y)$</p> <p>$\therefore \hat{BOC} = 2 \times \hat{BAC}$</p>	<p>✓ construction</p> <p>✓ S/R</p> <p>✓ S/R</p> <p>✓ S</p> <p>✓ deduction</p> <p>(5)</p>
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8.2



8.2.1	$\hat{D} = x$ (\angle opp – sides) $\hat{E}_2 = x$ (tan-chord theorem)	✓S ✓R ✓S ✓R (4)
8.2.2	$\hat{C}_2 = 2x$ (ext \angle of Δ) $2\hat{C}_2 = \hat{EOB} = 4x$ (\angle at centre = $2 \times \angle$ at circumf)	✓S/R ✓S ✓R (3)
8.2.3	$\hat{OBD} = 90^\circ$ (tan \perp rad) $\hat{B}_2 = 90^\circ - x$	✓S ✓R (2)
8.2.4	$\hat{OBD} = 90^\circ$ (tan \perp rad) $\hat{F}_2 = 180^\circ - x - \hat{OBD}$ (sum \angle s Δ) $\hat{F}_2 = 90^\circ - x$ $\therefore BC = FC$ (opp \angle 's =) $\therefore DC = BC = FC$	✓S ✓S ✓S/R (3)
		[17]

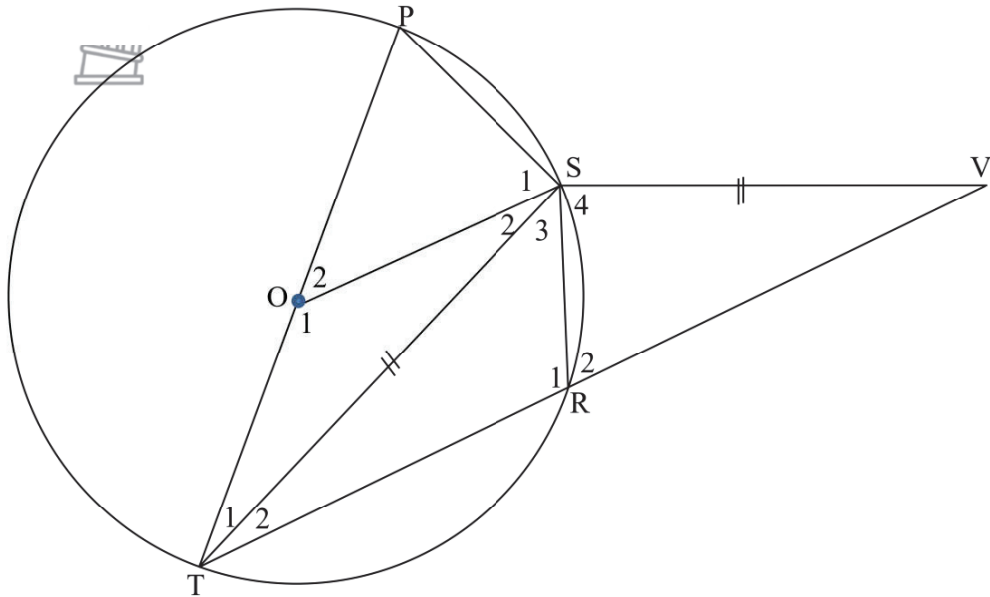
QUESTION 9



9.1	<p>In $\triangle BCA$ and $\triangle CDB$</p> $\frac{BC}{CD} = \frac{12}{18} = \frac{2}{3}$ $\frac{AB}{BC} = \frac{8}{12} = \frac{2}{3}$ $\frac{AC}{DB} = \frac{10}{15} = \frac{2}{3}$ <p>$\triangle BCA \parallel \triangle CDB$ (sides in proportion)</p>	<p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S/R</p> <p>(4)</p>
9.2	<p>$\angle \hat{B}C = \angle \hat{C}D$ ($\triangle BCA \parallel \triangle CDB$)</p> <p>$\therefore AB \parallel CD$ (alt \angles=)</p>	<p>✓ S</p> <p>✓ R</p> <p>(2)</p>
9.3	<p>In $\triangle ABE$ and $\triangle DCE$:</p> <p>$\angle \hat{B}C = \angle \hat{C}D$ (proven)</p> <p>$\angle \hat{E}B = \angle \hat{E}D$ (vertically opp \angle)</p> <p>$\triangle ABE \parallel \triangle DCE$ ($\angle \angle \angle$)</p> $\frac{AB}{DC} = \frac{BE}{CE} \quad (\parallel \Delta's)$ $\frac{8}{18} = \frac{x}{12-x}$ $8(12-x) = 18x$ $96 - 8x = 18x$ $96 = 26x$ $x = 3,69$ $CE = 8,31$	<p>✓ $\triangle ABE \parallel \triangle DCE$</p> <p>✓ $\frac{AB}{DC} = \frac{BE}{CE}$</p> <p>✓ $\frac{8}{18} = \frac{x}{12-x}$</p> <p>✓ $CE = 8,31$</p> <p>(4)</p>

		[10]
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QUESTION 10



10.1	$\hat{PST} = 90^\circ$ (\angle in semi-circle)	✓ S ✓ R (2)
10.2	$\hat{T}_1 = \hat{T}_2$ (ST is a bisector) $\hat{T}_2 = \hat{V}$ (\angle opp = sides) $\hat{T}_1 = \hat{V}$ $\hat{R}_2 = \hat{P}$ (ext \angle of cyclic quad) $\hat{S}_4 = \hat{PST} = 90^\circ$ (sum \angle of Δ)	✓ S ✓ S ✓ S ✓ R ✓ S + R (5)
10.3	$\Delta TSO \parallel \Delta TVS$ $\hat{T}_1 = \hat{T}_2$ (ST is a bisector) $\hat{S}_2 = \hat{V} = \left(\text{both} = \hat{T}_1 \right)$ $\hat{O}_1 = \hat{TSV}$ (3rd \angle Δ) $\Delta TSO \parallel \Delta TVS$ ($\angle \angle \angle$)	✓ S ✓ S ✓ S/R (3)

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10.4	$\frac{TS}{TV} = \frac{OS}{VS} \quad \Delta TSO \parallel \Delta TVS \quad (\angle\angle\angle)$ $TS.VS = OS.TV$ <p><i>but</i> $TS = VS$ <i>given</i></p> <p><i>and</i> $OS = \frac{1}{2}PT$ <i>both radii</i></p> $\therefore VS.VS = \frac{1}{2}PT.TV$ $\therefore 2VS^2 = PT.TV$	<p>✓ S</p> <p>✓ S</p> <p>✓ S+R</p> <p>✓ substitution</p> <p style="text-align: right;">(4)</p>
		[14]

TOTAL: 150



education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

**NATIONAL SENIOR CERTIFICATE/
*NASIONALE SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

SEPTEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

**These marking guidelines consist of 20 pages and a 3 page cognitive grid/
*Hierdie nasienriglyne bestaan uit 20 bladsye en 'n 3 bladsye kognitiewe tabel***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

QUESTION/VRAAG 1

63	79	50	74	75	66	150	86	72	74	60
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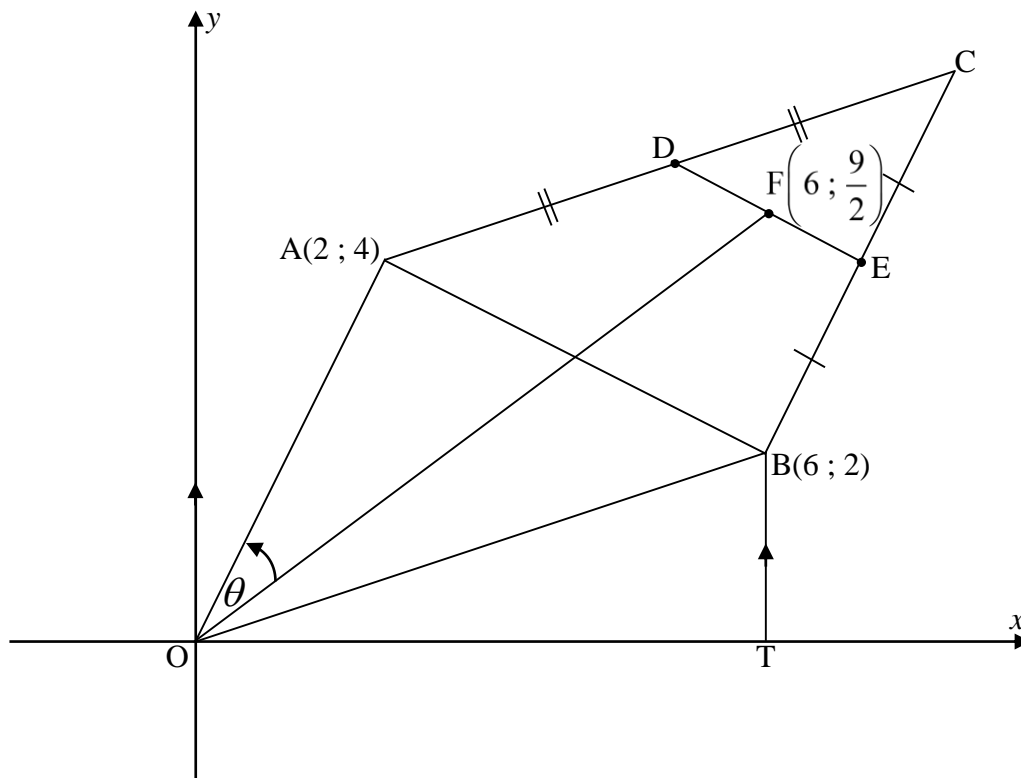
1.1.1	$\bar{x} = \frac{849}{11}$ $= 77,18$	Answer only: full marks	✓ 849 (addition of results/ <i>optel van uitslae</i>) ✓ answer/antwoord (CA if/as ÷ 11) (2)
1.1.2	$\sigma = 24,86$	No penalty for rounding:	✓ answer/antwoord (A) (1)
1.1.3	$(\bar{x} - \sigma ; \bar{x} + \sigma)$ $= (52,32 ; 102,04)$ \therefore 2 results/ <i>uitslae</i>	Answer only: full marks provided 1.1.1 & 1.1.2 both correct	✓ 52,32 ✓ 102,04 ✓ answer/antwoord (3)
1.2	150		✓ answer/antwoord (1)
1.3	D		✓✓ answer/antwoord (2)
1.4	By doubling any of the six lowest results, the new result will be more than 74 and lie to the right hand side of 74/ <i>Deur om enige uitslag van die ses laagste uitslae te verdubbel sal die nuwe uitslag meer as 74 wees en regs van 74 lê.</i> Because 74 is also the mode, the new median will still be 74/ <i>Omdat 74 die modus is sal die nuwe mediaan 74 bly.</i> The median therefore has not changed/ <i>Die mediaan het dus nie verander nie.</i>		✓ 74 mode/modus ✓ no change/geen verandering (2)
			[11]

QUESTION/VRAAG 2

Temperature at midday (in °C)/ Middaguur-temperatuur (in °C)	16	20	23	29	33	38	40	38	35	30
Number of ice creams / Aantal roomyse	12	17	19	44	64	70	74	66	60	40

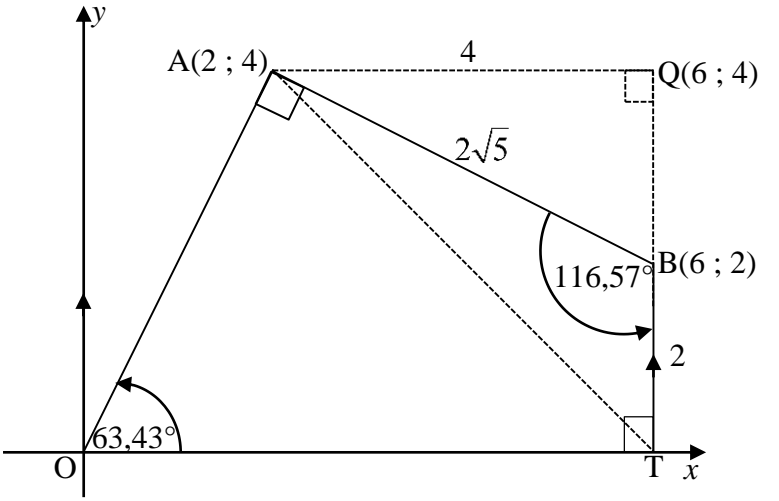
2.1	<p>Strong/Sterk The majority of the points lie close to the regression line/ <i>Die meerderheid punte lê naby die regressielyn</i></p> <p>OR/OF</p> <p>Strong/Sterk $r = 0,98$</p>	<p>✓ strong/sterk ✓ reason/rede (2)</p> <p>✓ strong/sterk ✓ $r = 0,98$ (2)</p>	
2.2	<p>$a = -38,4828\dots$ $b = 2,8173\dots$ $\hat{y} = -38,48 + 2,82x$</p>	<div style="border: 1px solid black; padding: 5px;"> <p>Answer only: full marks, but if a and b are swapped only 1/3 marks/ <i>maar as a en b omgeruil is, slegs 1/3 punte.</i></p> </div>	<p>✓ a ✓ b ✓ equation/vergelyking (3)</p>
2.3	<p>$\hat{y} = -38,48 + 2,82(26)$ $= 34,84$ $\therefore 34$ ice creams / roomyse</p> <p>OR/OF</p> <p>$26\hat{y} = 34,77$ (calculator / sakrekenaar) $\therefore 34$ ice cream / roomyse</p>	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>Answer only: full marks</p> </div> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"> <p>Can also accept 35</p> </div>	<p>✓ substitute 26 into eq. / vervang 26 in vgl. ✓ 34 (2)</p> <p>✓✓ 34 (2)</p>
2.4	<p>Regression line will be pulled slightly upwards/<i>regressielyn sal effe opwaarts getrek word</i></p> <p>The prediction will be that more ice cream will be sold/ <i>Die voorspelling is dat meer roomyse verkoop sal word.</i></p>	<p>✓ explanation/verduideliking ✓ more ice cream/meer roomyse (2)</p>	
		[9]	

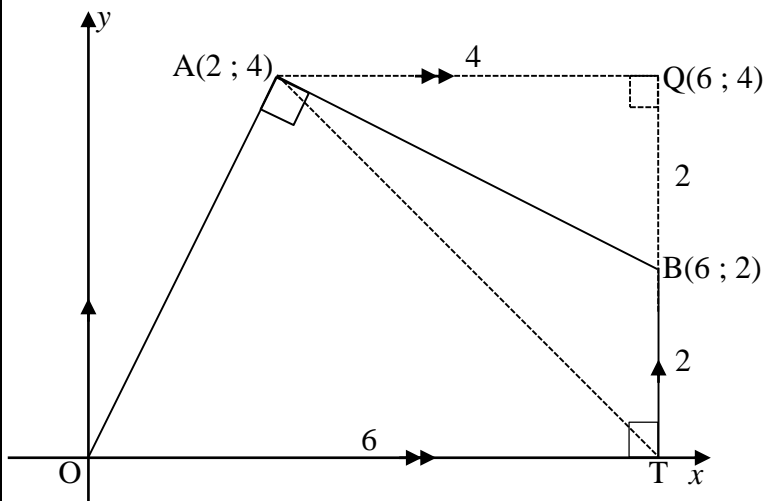
QUESTION/VRAAG 3



3.1.1	$AB = \sqrt{(6 - 2)^2 + (2 - 4)^2}$ $= 2\sqrt{5}$	✓ subst./vervang ✓ answer/antwoord (2)
3.1.2	$m_{AB} = \frac{2 - 4}{6 - 2}$ $= -\frac{1}{2}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px auto;">Answer only: full marks</div>	✓ subst./vervang ✓ answer/antwoord (2)
3.2	$m_{OA} = 2$ But / maar : $m_{OA} \times m_{AB}$ $= 2 \left(-\frac{1}{2} \right)$ $= -1$ $\therefore OA \perp AB$	✓ $m_{OA} = 2$ $\checkmark 2 \left(-\frac{1}{2} \right) = -1$ (2)

<p>3.3</p>	<p>DE AB [midpt.theorem / <i>midpt. – stelling</i>]</p> $\therefore m_{DE} = -\frac{1}{2}$ <p>through / <i>deur</i> $\left(6; \frac{9}{2}\right)$</p> $y - \frac{9}{2} = -\frac{1}{2}(x - 6)$ $\therefore y = -\frac{1}{2}x + \frac{15}{2}$	<p>✓ S OR/OF R</p> <p>✓ m_{DE}</p> <p>✓ subst./<i>vervang</i> $\left(6; \frac{9}{2}\right)$</p> <p>✓ equation/ <i>vergelyking</i></p> <p>(4)</p>
<p>3.4</p>	<p>If / <i>As</i> AOBC is parm: Answer only: full marks</p> <p>OA BC and / <i>en</i> OB AC</p> <p>$x_O \rightarrow x_B = x_O + 6$ (translation / <i>translasie</i>)</p> <p>$\therefore x_A \rightarrow x_C = 2 + 6$</p> <p>$\therefore x_C = 8$</p> <p>in the same way / <i>op dieselfde wyse</i>:</p> <p>$y_A \rightarrow y_C = 4 + 2$</p> <p>$\therefore y_C = 6$</p> <p>$\therefore C(8; 6)$</p> <p>OR/OF</p> <p>midpt. AB = (4 ; 3)</p> <p>\therefore midpt. OA = (4 ; 3) [diag./ <i>hoekln. parm.</i>]</p> <p>$\frac{0 + x_C}{2} = 4$ and / <i>en</i> $\frac{0 + y_C}{2} = 3$</p> <p>$\therefore x_C = 8$ $\therefore y_C = 6$</p> <p>$\therefore C(8; 6)$</p> <p>OR/OF</p> <p>eq./ <i>vgl.</i> BC: $y = 2x - 10$</p> <p>eq./ <i>vgl.</i> AC: $y = \frac{1}{3}x + \frac{10}{3}$</p> <p>$2x - 10 = \frac{1}{3}x + \frac{10}{3}$</p> <p>$5x = 40$</p> <p>$x = 8$</p> <p>and / <i>en</i>: $y = 2(8) - 10 = 6$</p> <p>$\therefore C(8; 6)$</p>	<p>✓ method/<i>metode</i></p> <p>✓ $x_C = 8$</p> <p>✓ $y_C = 6$</p> <p>(3)</p> <p>✓ method/<i>metode</i></p> <p>✓ $x_C = 8$</p> <p>✓ $y_C = 6$</p> <p>(3)</p> <p>✓ method/<i>metode</i></p> <p>✓ $x_C = 8$</p> <p>✓ $y_C = 6$</p> <p>(3)</p>

<p>3.5.1</p>	$\tan \hat{A}OT = 2$ $\therefore \hat{A}OT = 63,43^\circ$ $\tan \hat{F}OT = \frac{3}{4}$ $\therefore \hat{F}OT = 36,87^\circ$ $\theta = 63,43^\circ - 36,87^\circ$ $\therefore \theta = 26,56^\circ$	<p>✓ $\tan \hat{A}OT = m_{OA}$</p> <p>✓ answer/antwoord</p> <p>✓ $\tan \hat{F}OT = \frac{3}{4}$</p> <p>✓ answer/antwoord</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(5)</p>
<p>3.5.2</p>	 <p>BT = 2 units/eenhede</p> <p>$\perp_h = 4$</p> $\text{area } \triangle ABT = \frac{1}{2}(2)(4)$ $= 4 \text{ units}^2 / \text{eenhede}^2$ <p>OR/OF</p> <p>BT = 2 units/eenhede</p> <p>$\hat{A}BT = 116,57^\circ$ [\angles of quad./ \anglee van vierhk.]</p> $\text{area } \triangle ABT = \frac{1}{2}(2\sqrt{5})(2)\sin 116,57^\circ$ $= 3,9998\dots$ $\approx 4 \text{ units}^2 / \text{eenhede}^2$ <p>OR/OF</p>	<p>✓ BT = 2</p> <p>✓ $\perp_h = 4$</p> <p>✓ subst./vervanging</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(4)</p> <p>✓ BT = 2</p> <p>✓ $\hat{A}BT$</p> <p>✓ subst./vervanging</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(4)</p>



$$\begin{aligned} \text{Area } \triangle ABT &= \text{Area AOTQ} - \text{Area } \triangle AOT - \text{Area } \triangle ABQ \\ &= \frac{1}{2}(4 + 6) \times 4 - \frac{1}{2}(6)(4) - \frac{1}{2}(2)(4) \\ &= 4 \text{ units}^2 / \text{eenhede}^2 \end{aligned}$$

✓ method/metode

$$\checkmark \frac{1}{2}(4 + 6) \times 4$$

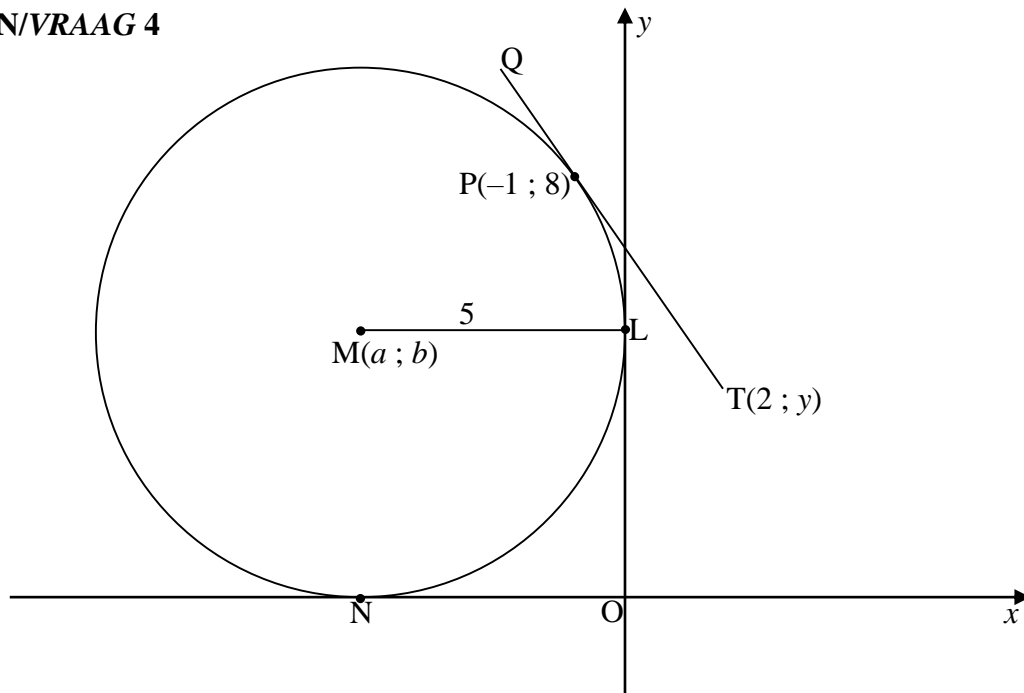
$$\checkmark \frac{1}{2}(6)(4) \& \frac{1}{2}(2)(4)$$

✓ answer/antwoord

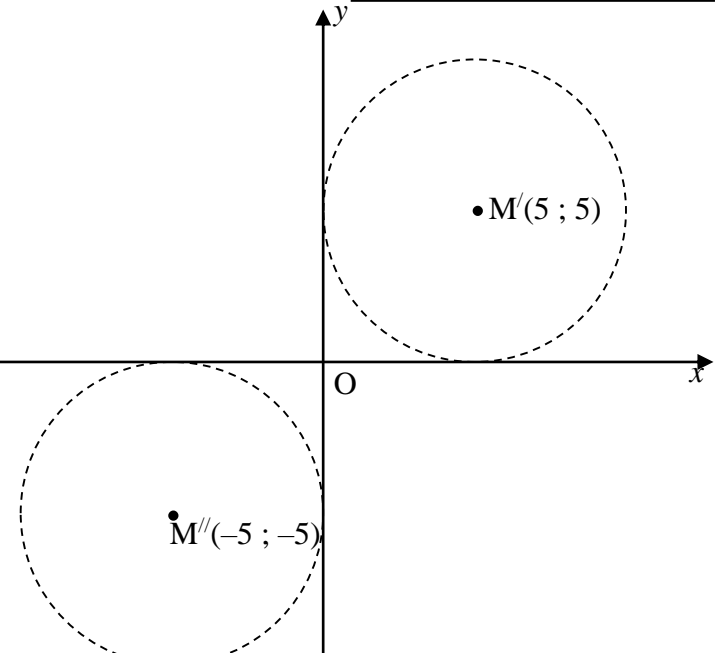
(4)

[22]

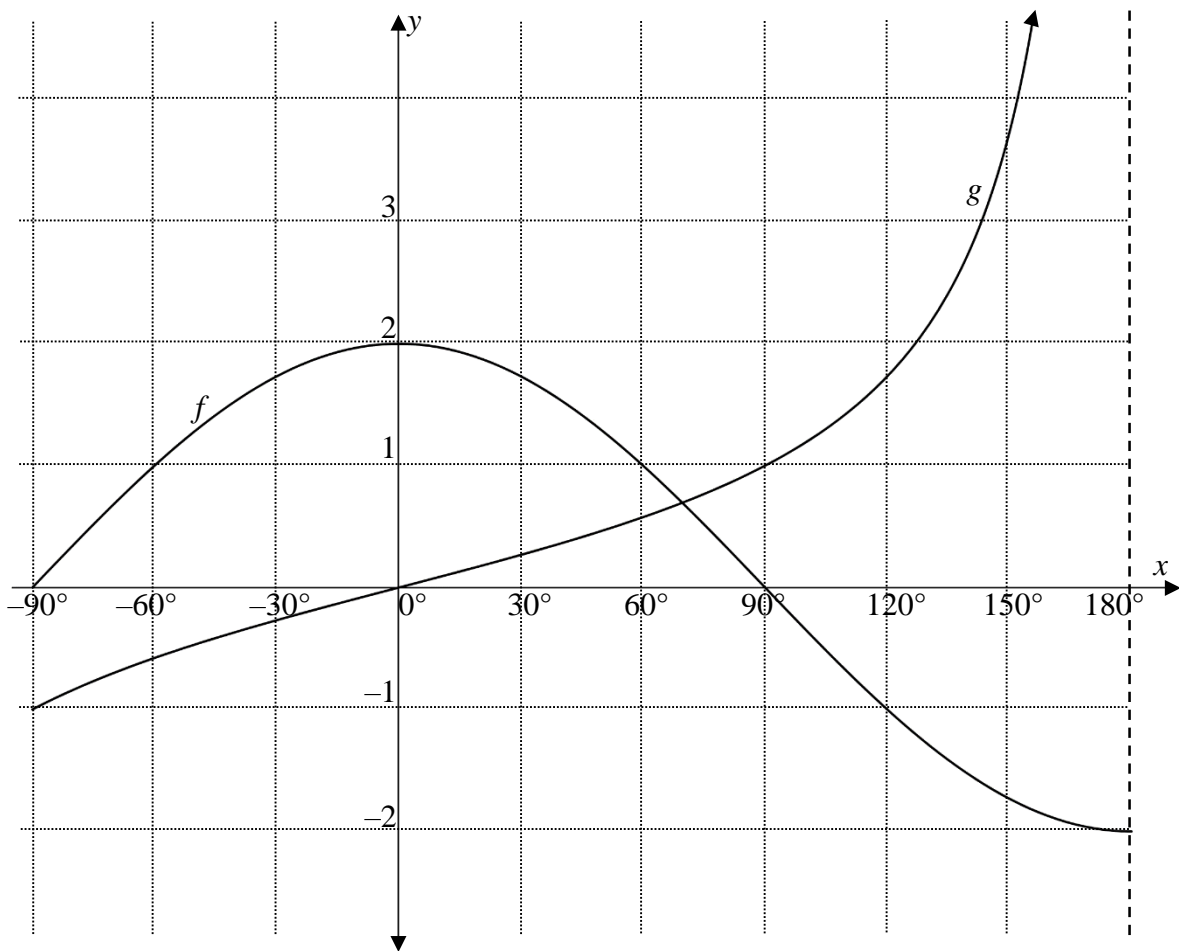
QUESTION/VRAAG 4



4.1	radius \perp tangent / <i>radius \perp raaklyn</i>	\checkmark rede/reason (1)
4.2.1	M(-5 ; 5)	$\checkmark x = -5$ $\checkmark y = 5$ (2)
4.2.2	$(x + 5)^2 + (y - 5)^2 = 25$	$\checkmark (x + 5)^2 + (y - 5)^2$ $\checkmark r^2 = 25$ (2)
4.2.3	$m_{MP} = \frac{8-5}{-1-(-5)}$ $= \frac{3}{4}$ $\therefore m_{QT} = -\frac{4}{3} \text{ [rad } \perp \text{ tangent / rad } \perp \text{ raaklyn]}$ $8 = -\frac{4}{3}(-1) + c \quad \text{OR / OF} \quad y - 8 = -\frac{4}{3}(x - (-1))$ $c = \frac{20}{3}$ $y = -\frac{4}{3}x + \frac{20}{3}$	\checkmark subst./vervanging $\checkmark m_{MP} = \frac{3}{4}$ $\checkmark m_{QT} = -\frac{4}{3}$ \checkmark subst./vervang m & $(-1 ; 8)$ \checkmark equation / vergelyking (5)

<p>4.3</p>	$y = -\frac{4}{3}(2) + \frac{20}{3}$ $y = 4$ $\therefore T(2;4)$ $MT = \sqrt{(2 - (-5))^2 + (4 - 5)^2}$ $= 5\sqrt{2}$ <p>radius circle T / radius sirkel T = $5\sqrt{2} - 5$</p> $(x - 2)^2 + (y - 4)^2 = 75 - 50\sqrt{2}$ $\approx 4,29$	<p>✓ subst./vervanging</p> <p>✓ $y_T = 4$</p> <p>✓ subst./vervanging</p> <p>✓ answer/antwoord</p> <p>✓ $MT - 5$</p> <p>✓ equation / vergelyking</p> <p style="text-align: right;">(6)</p>
<p>4.4</p>	<p>If the x-axis and y-axis have to remain simultaneously as tangents to the circle M, then the circle is reflected across the axes/As die x-as en y-as gelyktydig as raaklyne aan die sirkel M bly, dan word die sirkel oor die asse gereflekteer.</p> <p>The only quadrants where $xy \geq 0$, are quadrants 1 and 3/ Die enigste kwadrante waar $xy \geq 0$, is kwadrante 1 en 3</p> <p>$M'(5 ; 5)$ & $M''(-5 ; -5)$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Accuracy /Akkuraatheid But CA from 4.2.1 / Maar CA van 4.2.1</p> </div> 	<p>✓✓ $M'(5 ; 5)$</p> <p>✓✓ $M''(-5 ; -5)$</p> <p style="text-align: right;">(4)</p>
<p>[20]</p>		

QUESTION/VRAAG 5



5.1	$b = \frac{1}{2}$	✓ answer/antwoord (1)
5.2	$y \geq -1$ OR/OF $y \in [-1; \infty)$	✓ critical values/kritiese waardes ✓ notation/notasie (2)
5.3	360°	✓ 360° (1)
5.4	$x = 85^\circ$	✓ 85° (1)
5.5	$x = 0^\circ$ or/of $x = 180^\circ$	✓ 0° ✓ 180° (2)
5.6	$p = 180^\circ$	✓✓ 180° (2)
		[9]

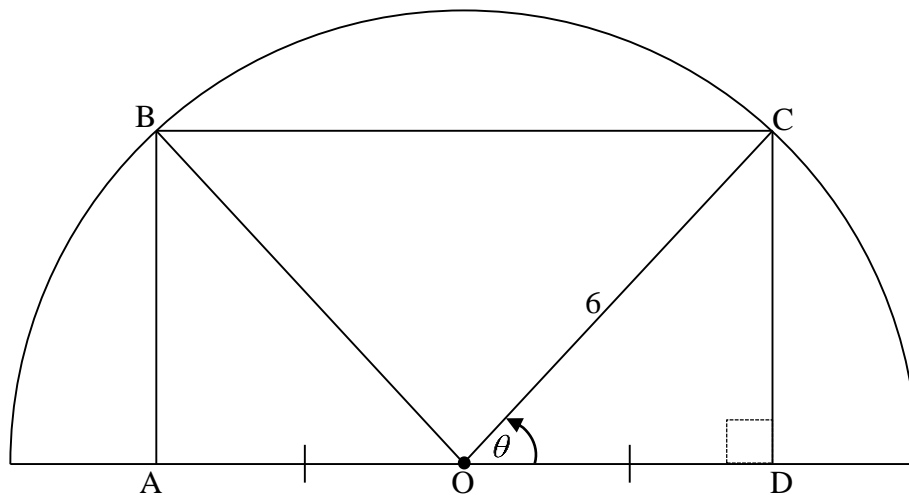
QUESTION/VRAAG 6

6.1.1	$\sin 335^\circ$ $= -\sin 25^\circ$	$\checkmark -\sin 25^\circ$ <p style="text-align: right;">(1)</p>
6.1.2	$\cos 50^\circ$ $= \cos 2(25^\circ)$ $= 1 - 2\sin^2 25$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Answer only: full marks</div>	$\checkmark \cos 2(25^\circ)$ $\checkmark 1 - 2\sin^2 25$ <p style="text-align: right;">(2)</p>
6.2	$\frac{\sin(-2x) \cdot (1 - \sin^2 x)}{\sin(90^\circ + x) \cdot \tan x}$ $= \frac{(-\sin 2x)(\cos^2 x)}{(\cos x)\left(\frac{\sin x}{\cos x}\right)}$ $= \frac{-2\sin x \cdot \cos x \cdot \cos^2 x}{\sin x}$ $= -2\cos^3 x$	$\checkmark -\sin 2x$ $\checkmark \cos^2 x$ $\checkmark \cos x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark -2\sin x \cdot \cos x$ $\checkmark -2\cos^3 x$ <p style="text-align: right;">(6)</p>
6.3	$(p \tan 30^\circ + q \sin 60^\circ)^2$ $= \left(p \cdot \frac{1}{\sqrt{3}} + q \cdot \frac{\sqrt{3}}{2}\right)^2$ $= \left(\frac{p}{\sqrt{3}} + \frac{\sqrt{3}q}{2}\right)^2$ $= \frac{p^2}{3} + pq + \frac{3q^2}{4}$ $= \frac{4p^2 + 12pq + 9q^2}{12}$	$\checkmark \text{substitution/vervanging}$ $\checkmark \text{expansion/uitbreiding}$ $\checkmark \text{answer/antwoord}$ <p style="text-align: right;">(3)</p>

6.4.1	$\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ + B) - A]$ $= \cos(90^\circ + B) \cdot \cos A + \sin(90^\circ + B) \cdot \sin A$ $= (-\sin B) \cdot \cos A + \cos B \cdot \sin A$ $= \sin A \cdot \cos B - \sin B \cdot \cos A$ <p>OR/OF</p> $\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ - A) - (-B)]$ $= \cos(90^\circ - A) \cdot \cos(-B) + \sin(90^\circ - A) \cdot \sin(-B)$ $= \sin A \cdot \cos B + \cos A \cdot (-\sin B)$ $= \sin A \cdot \cos B - \sin B \cdot \cos A$	<p>✓ co-ratio/ko-verhouding</p> <p>✓ writing as difference of A and B / skryf as die verskil van A en B</p> <p>✓ expansion/ uitbreiding</p> <p>✓ all reductions/alle reduksies</p> <p>(4)</p> <p>✓ co-ratio/ko-verhouding</p> <p>✓ writing as difference of A and B / skryf as die verskil van A en B</p> <p>✓ expansion/ uitbreiding</p> <p>✓ all reductions/alle reduksies</p> <p>(4)</p>
6.4.2	$\text{LHS/LK} = \sin 9A + \sin A$ $= \sin(5A + 4A) + \sin(5A - 4A)$ $= \sin 5A \cdot \cos 4A + \sin 4A \cdot \cos 5A + \sin 5A \cdot \cos 4A - \sin 4A \cdot \cos 5A$ $= 2\sin 5A \cdot \cos 4A$	<p>✓ $\sin(5A + 4A)$</p> <p>✓ $\sin(5A - 4A)$</p> <p>✓ expansion/ uitbreiding</p> <p>(3)</p>
6.4.3	<p>Max value of/Maks waarde van: $2 \sin 5A \cdot \cos 4A = 2$</p> <p>$\therefore 3^2 = 9$</p> <p>$\therefore$ max value / maks. waarde : $3^{2\sin 5A \cdot \cos 4A} = 9$</p>	<p>✓ max value/maks. waarde</p> <p>$2 \sin 5A \cdot \cos 4A = 2$</p> <p>✓ 9</p> <p>(2)</p>

<p>6.5</p>	$\cos 2x - 5 \cos x - 2 = 0$ $2 \cos^2 x - 1 - 5 \cos x - 2 = 0$ $2 \cos^2 x - 5 \cos x - 3 = 0$ $(2 \cos x + 1)(\cos x - 3) = 0$ $\cos x = -\frac{1}{2} \text{ or / of } \cos x = 3$ $\text{ref. } \angle = 60^\circ \quad \cos x \neq 3$ $x = 120^\circ + k.360^\circ ; \text{ or / of } x = 240^\circ + k.360^\circ ; k \in Z$	<p>✓ $2 \cos^2 x - 1$</p> <p>✓ factors/faktore</p> <p>✓ both equations/beide vergls.</p> <p>✓ $\cos x \neq 3$</p> <p>✓ $x = 120^\circ$ & $x = 240^\circ$</p> <p>✓ $+ k.360^\circ, k \in Z$</p> <p style="text-align: right;">(6)</p>
<p>6.6</p>	$\tan x = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$ $\tan^2 x = \sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$ $\tan^2 x = \sin x + \tan x$ $\frac{\sin^2 x}{\cos^2 x} = \sin x + \frac{\sin x}{\cos x}$ $\sin^2 x = \sin x \cdot \cos^2 x + \sin x \cdot \cos x$ $2 \sin^2 x = 2 \sin x \cdot \cos^2 x + 2 \sin x \cdot \cos x$ $2 \sin^2 x = \sin 2x \cdot \cos x + \sin 2x$ $2 \sin^2 x = \sin 2x (\cos x + 1)$	<p>✓</p> $\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x \dots}}}$ <p>✓ $\tan^2 x = \sin x + \tan x$</p> <p>✓ $\frac{\sin^2 x}{\cos^2 x} = \sin x + \frac{\sin x}{\cos x}$</p> <p>✓ multiply by/vermenigvuldig met $2 \cos^2 x$</p> <p>✓ double angle identity/dubbelhoekidentiteit</p> <p style="text-align: right;">(5)</p>
[32]		

QUESTION/VRAAG 7

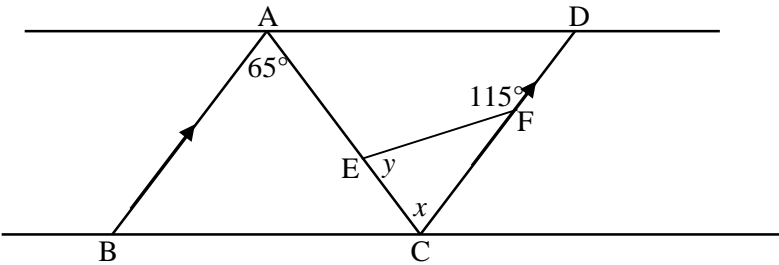


7.1	$\hat{B}OA = \theta$ $\therefore \hat{B}OC = 180^\circ - 2\theta$	$\checkmark \hat{B}OA = \theta$ \checkmark answer/antwoord (2)
7.2	$\hat{B}OC = 94^\circ$ $BC^2 = OB^2 + OC^2 - 2OB \cdot OC \cdot \cos \hat{B}OC$ $BC^2 = 6^2 + 6^2 - 2(6)(6)\cos 94^\circ$ $BC^2 = 77,0224\dots$ $\therefore BC = 8,78$ units / eenhede	$\checkmark \hat{B}OC$ \checkmark Subst. in cosine rule correctly/vervang korrek in cos-reël \checkmark answer/antwoord (3)
7.3	For ABCD to be a square/Vir ABCD om vierkant te wees: $AD = DC$ $\therefore DC = 2OD$ In $\triangle ODC$ $\tan \theta = \frac{DC}{OD}$ $= \frac{2OD}{OD}$ $= 2$ $\therefore \theta = \tan^{-1}(2)$ $\theta = 63,43^\circ$	$\checkmark DC = 2OD$ $\checkmark \tan \theta = \frac{DC}{OD}$ $\checkmark \tan \theta = 2$ $\checkmark \theta = 63,43^\circ$ (4)
		[9]

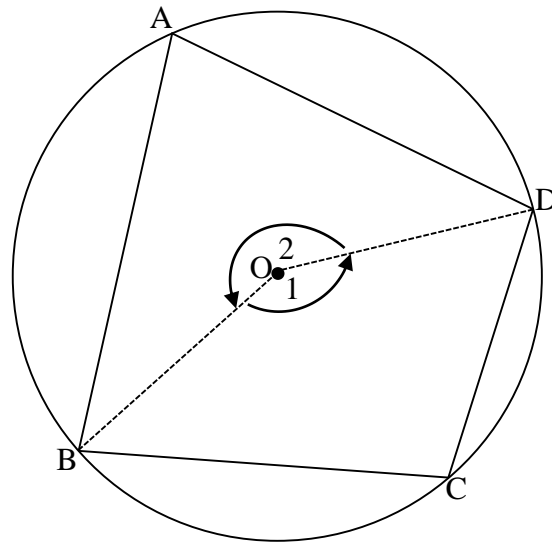
GEOMETRY/MEETKUNDE

Please read carefully through the following table before marking **QUESTION 8–10/**

*Lees asseblief sorgvuldig deur die volgende tabel alvorens **VRAAG 8–10** nagesien word.*

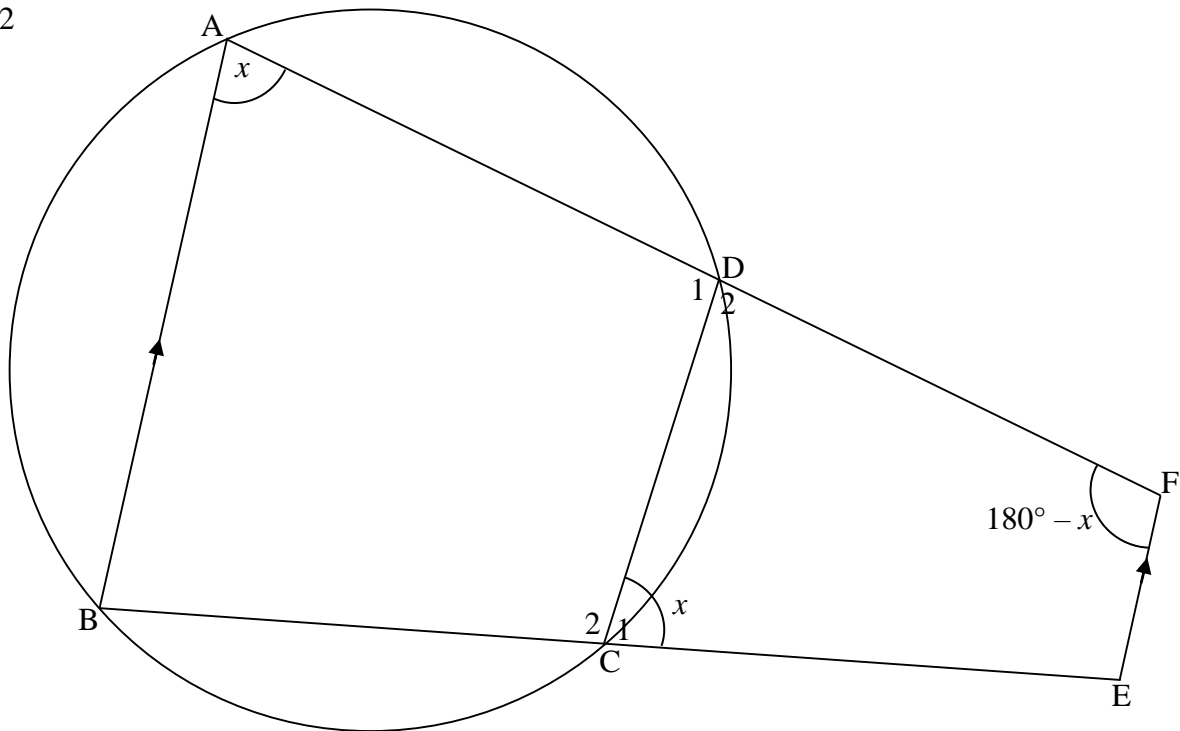
	<p>The order in which the candidate answers a geometry question must follow logically/<i>Die volgorde waarin 'n kandidaat 'n meetkundevraag beantwoord moet logies volg.</i></p> <p>Example/Voorbeeld</p> <p>Given/<i>Gegee</i> $AB \parallel CD$ and/<i>en</i> $\hat{EFD} = 115^\circ$</p>  <p>The candidate first needs to calculate x BEFORE he/she can calculate y/<i>Die kandidaat moet eerste vir x bereken VOORDAT hy/sy vir y kan bereken.</i></p>
S	<p>A mark for a correct statement (A statement mark is independent of a reason) <i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i></p>
R	<p>A mark for the correct reason (A reason mark may only be awarded if the statement is correct) <i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i></p>
S/R	<p>Award a mark if the statement AND reason are both correct (Both MUST be correct to get one mark) <i>Ken 'n punt toe as die bewering EN rede beide korrek is (Beide MOET korrek wees om een punt te kry)</i></p>

QUESTION/VRAAG 8



<p>8.1</p>	<p>Construction/<i>Konstr.</i>: Draw BO and OD/<i>Trek BO en OD.</i></p> <p>$\hat{O}_1 = 2\hat{A}$ [\angle at centre = $2 \times \angle$ at circ./ <i>midpts</i> $\angle = 2 \times$ <i>omtreks</i> \angle]</p> <p>$\hat{O}_2 = 2\hat{C}$ [\angle at centre = $2 \times \angle$ at circ./ <i>midpts</i> $\angle = 2 \times$ <i>omtreks</i> \angle]</p> <p>$\hat{O}_1 + \hat{O}_2 = 360^\circ$ [\angles around a point/\anglee om 'n punt]</p> <p>$\therefore 2\hat{A} + 2\hat{C} = 360^\circ$</p> <p>$\therefore 2(\hat{A} + \hat{C}) = 360^\circ$</p> <p>$\therefore \hat{A} + \hat{C} = 180^\circ$</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>The last statement mark / <i>Die laaste punt vir bewering:</i></p> <p>$2\hat{A} + 2\hat{C} = 360^\circ$</p> <p>OR / OF</p> <p>$2(\hat{A} + \hat{C}) = 360^\circ$</p> </div>	<p>✓ constr./ <i>konstr.</i></p> <p>✓ S/R</p> <p>✓ S</p> <p>✓ S/R</p> <p>✓ S</p> <p>(5)</p>
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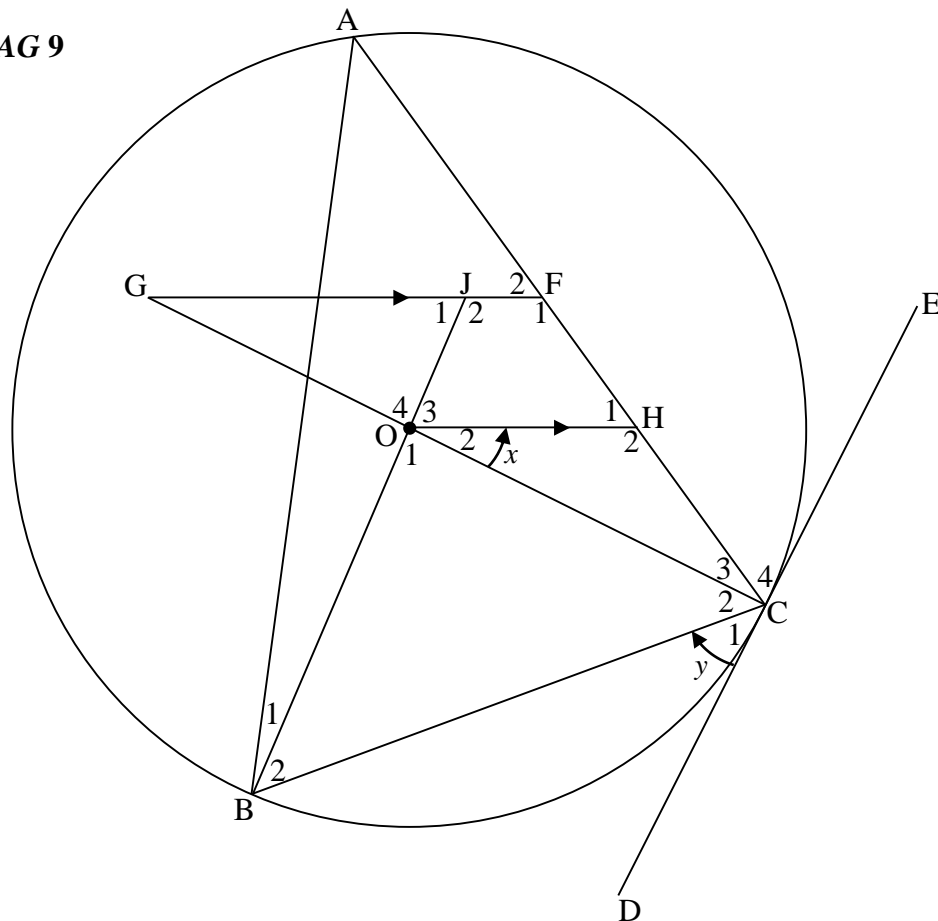
8.2



Let/Laat: $\hat{A} = x$

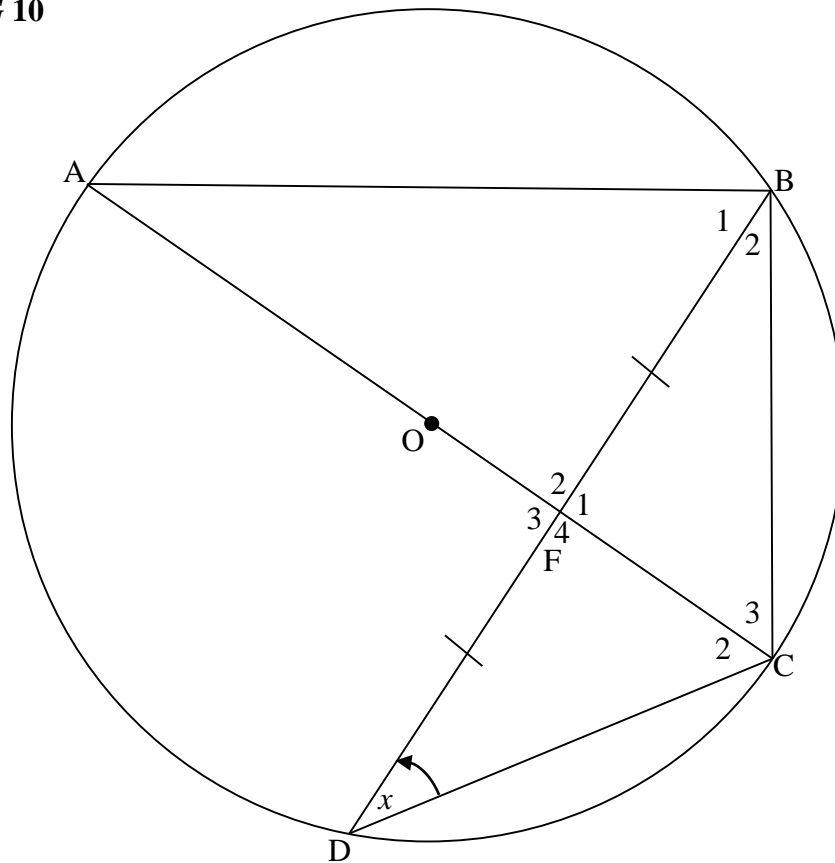
<p>8.2</p>	<p>$\hat{C}_1 = \hat{A} = x$ [ext. \angle cyclic quad./buite \angle kvh.] $\hat{F} = 180^\circ - \hat{A} = 180^\circ - x$ [co-interior \angles/ko-binne \anglee; $AB \parallel FE$] $\hat{C}_1 + \hat{F} = x + 180^\circ - x = 180^\circ$ \therefore CEFD is a cyclic quad/is 'n kvh. [converse opp. \angles cyclic quad/omgekeerde teenorst. \anglee kvh] / [ext. \angle of quad = opp. interior \angle/buite \angle van vierhoek = teenorst. binne \angle]</p> <p>OR / OF</p> <p>$\hat{C}_2 = 180^\circ - \hat{A} = 180^\circ - x$ [opp. \angles cyclic quad/teenorst. \anglee kvh] $\hat{F} = 180^\circ - \hat{A} = 180^\circ - x$ [co-interior \angles/ko-binne \anglee; $AB \parallel FE$] $\hat{C}_2 = \hat{F} = 180^\circ - x$ \therefore CEFD is a cyclic quad/is 'n kvh. [converse ext. \angle cyclic quad/omgekeerde buite. \angle kvh] / [opp. \angles of quad supplementary/teenorst. \anglee van vierhoek supplementêr]</p>	<p>✓S✓R ✓S/R ✓S ✓R (5)</p> <p>✓S✓R ✓S/R ✓S ✓R (5)</p>
		<p>[10]</p>

QUESTION/VRAAG 9



<p>9.1</p>	<p>$\hat{C}_1 = \hat{A} = y$ [tan-chord theorem/raaklyn - koordstelling] $\hat{O}_1 = 2\hat{A} = 2y$ [\angle at centre = $2 \times \angle$ at circ./ midpts $\angle = 2 \times$ omtreks \angle] $\hat{O}_3 = 180^\circ - 2y - x$ [\angles on straight line / \anglee op reguit lyn] $\hat{J}_1 = 180^\circ - 2y - x$ [corresponding \angles / ooreenkomstige \anglee; GF \parallel OH]</p> <p>OR/OF</p> <p>$\hat{C}_1 = \hat{A} = y$ [tan-chord theorem/raaklyn - koordstelling] $\hat{O}_1 = 2\hat{A} = 2y$ [\angle at centre = $2 \times \angle$ at circ./ midpts $\angle = 2 \times$ omtreks \angle] $\hat{G} = x$ [corresponding \angles / ooreenkomstige \anglee; GF \parallel OH] $\hat{O}_4 = 2y$ [opp. \angles = / regoorst. \anglee =] $\hat{J}_1 = 180^\circ - 2y - x$ [sum of \angles in Δ / som van \anglee in Δ]</p>	<p>✓S✓R ✓S✓R ✓S/R ✓S/R (6)</p> <p>✓S✓R ✓S✓R ✓S/R ✓S/R (6)</p>
<p>9.2</p>	<p>$\frac{FH}{FC} = \frac{GO}{GC}$ [prop.th / eweredigh.st ; GF \parallel OH] $= \frac{2}{5}$</p>	<p>✓S✓R ✓ answer/ antwoord (3)</p>
		<p>[9]</p>

QUESTION/VRAAG 10



10.1.1	$\hat{A} = x$ [\angle s in same segm./ \angle e in dies. segm.]	✓S ✓R (2)
10.1.2	$\hat{A}BC = 90^\circ$ [\angle in semi circle./ \angle in halvesirkel] $\hat{C}_3 = 90^\circ - x$ [sum of \angle s of Δ /som van \angle e van Δ]	✓S ✓R ✓S/R (3)
10.2.1	$OF \perp DB$ [line from centre to midpt.chord/midpt.sirkel, midpt.koord] $\therefore \hat{F}_2 = \hat{F}_3 = 90^\circ$	✓S ✓R (2)
10.2.2	In ΔCFB and / en ΔCBA $\hat{C}_3 = \hat{C}_3$ [common/gemeenskaplik] $\hat{F}_1 = \hat{B} = 90^\circ$ [from 10.1.2 & 10.2.1] $\therefore \Delta CFB \parallel \Delta CBA$ [$\angle \angle \angle$] OR/OF In ΔCFB and / en ΔCBA $\hat{C}_3 = \hat{C}_3$ [common/gemeenskaplik] $\hat{F}_1 = \hat{B} = 90^\circ$ [from 10.1.2 & 10.2.1] $\hat{B}_2 = \hat{A}$ [sum of \angle s of Δ /som van \angle e van Δ] $\therefore \Delta CFB \parallel \Delta CBA$	✓S/R ✓S ✓R (3) ✓S/R ✓S ✓S/R (3)

10.2.3	$\frac{CF}{CB} = \frac{CB}{CA} \text{ [from / vanuit } \triangle \text{]}$ $\therefore CB^2 = AC \cdot FC$ <p>but / maar : $\triangle DFC \equiv \triangle FBC$ [S \sphericalangle S]</p> $\therefore DC = BC$ $\therefore DC^2 = AC \cdot FC$	✓S/R ✓S ✓S/R ✓S (4)
10.2.4	$AC^2 = AB^2 + BC^2 \text{ [Pythagoras]}$ $AC^2 = AB^2 + FC \cdot AC \text{ [} BC^2 = DC^2 \text{]}$ $1 = \frac{AB^2}{AC^2} + \frac{FC}{AC}$ $\frac{FC}{AC} = 1 - \frac{AB^2}{AC^2}$ $\frac{FC}{AC} = \left(1 - \frac{AB}{AC}\right) \left(1 + \frac{AB}{AC}\right)$ <p>but / maar $AC = AO + OC$</p> $\therefore \frac{FC}{AC} = \left(1 - \frac{AB}{AO + OC}\right) \left(1 + \frac{AB}{AO + OC}\right)$	✓S ✓ $BC^2 = FC \cdot AC$ ✓ $\div AC^2$ ✓ $\frac{FC}{AC} = 1 - \frac{AB^2}{AC^2}$ ✓ factorise/faktoriseer (5)
		[19]

TOTAL/TOTAAL: 150

NSC/NSS – Cognitive grid/Kognitiewe tabel

GRID-Analysis (According to BLOOMS TAXONOMY)													
Question	Description	KNOWLEDGE			ROUTINE PROCEDURES			COMPLEXED PROCEDURES			PROBLEM SOLVING		
		Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
	Statistics(Gr.11) [11]												
1.1.1	Mean	2											
1.1.2	Standard deviation (calculator)				1								
1.1.3	Outside one SD					3							
1.2	Outlier	1											
1.3	Complex range							2					
1.4	Critical thinking median										2		
2	Regression(Gr.12) [9]												
2.1	Interpret scatter plot + regr. line			2									
2.2	Equation of regression line				3								
2.3	Estimation by using regr. line	2											
2.4	Critical thinking SD										2		
3	Analytical (Gr.11) [22]												
3.1.1	Distance		2										
3.1.2	Gradient		2										
3.2	Prove lines perpendicular				2								
3.3	Equation str.line + midpt.th							4					
3.4	4 th coordinate of parm						3						
3.5.1	Inclination							5					
3.5.2	Integrating trig to find area Δ								4				
4	Analytical(Gr.12) [20]												
4.1	Geometry theorem			1									
4.2.1	Coordinates centre			1	1								
4.2.2	Eq. of circle			2									
4.2.3	Equation of tangent					5							
4.3	Eq. of new circle with specific conditions									6			
4.4	Critical thinking new equation											4	

Question	Description	KNOWLEDGE			ROUTINE PROCEDURES			COMPLEX PROCEDURES			PROBLEM SOLVING		
		Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
5	Trig graphs [9]												
5.1	Parameters			1									
5.2	Range				2								
5.3	Period	1											
5.4	Translation					1							
5.5	Undefined							2					
5.6	Critical thinking + integrating concepts											2	
6	Trigonometry [32]												
6.1.1	Reduction	1											
6.1.2	Double angle			2									
6.2.	Simplification + combination of identities		2					4					
6.3	Special angles + combining algebra			2			1						
6.4.1	Derive formula $\sin(A - B)$				4								
6.4.2	Identity						3						
6.4.3	Max value										2		
6.5	General solution						6						
6.6	Critical thinking + evaluation												5
7	2D/3D Trig [9]												
7.1	Angle in terms of θ			2									
7.2.	Cosine rule						3						
7.3	Transforming a quadrilateral									4			
8	Geometry [10]												
8.1	Theorem				5								
8.2	Prove quad. to be cyclic quad.							5					

NSC/NSS – Cognitive grid/Kognitiewe tabel

Question	Description	KNOWLEDGE			ROUTINE PROCEDURES			COMPLEX PROCEDURES			PROBLEM SOLVING		
		Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
9	Geometry [9]												
9.1	Combinations of theorems							6					
9.2	Proportionality theorem					3							
10	Geometry [19]												
10.1.1	Angles in same segment	2											
10.1.2	Angle in semi-circle & angles of triangle			3									
10.2.1	Midpt.circle ; midpt. chord				2								
10.2.2	Similar triangles					3							
10.2.3	Combination of concepts							4					
10.2.4	Manipulation of sides										5		
		9	6	16	20	18	13	30	6	10	11	6	5

% Breakdown of cognitive levels	31	20,67%		51	34%		46	30,7%		22	14,67%
Expected %		20%			35%			30%			15%



DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

**PROVINCIAL PREPARATORY EXAMINATION/
PROVINSIALE VOORBEREIDINGSEKSAMEN**

GRADE/GRAAD 12

**MATHEMATICS P2/WIKUNDE V2
MARKING GUIDELINES/NASIENRIGLYNE
SEPTEMBER 2023**

MARKS/PUNTE: 150

**These marking guidelines consist of 22 pages/
*Hierdie nasienriglyne bestaan uit 22 bladsye.***

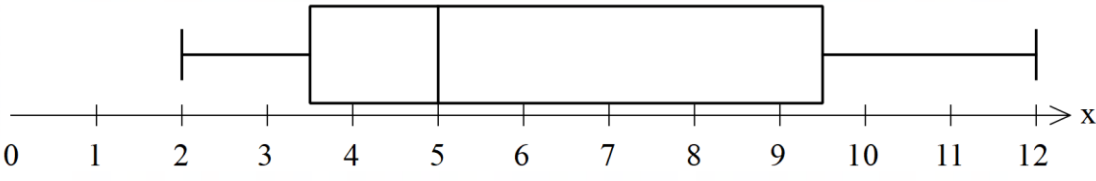
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
S/R	Award a mark if statement AND reason are both correct

QUESTION 1

2 3 4 5 5 8 9 10 12

1.1		✓ Median ✓ Min + Max ✓ Q2, Q3 (3)
1.2	Skew to the right.	✓ answer (1)
1.3	6,44	✓✓ answer (2)
1.4	$\sigma = 3,24$ $[6,44 - 3,24 ; 6,44 + 3,24]$ $[3,2 ; 9,68]$ 4 values lie outside / 4 waardes lê buite Round or square is o.k. Interested in the interval AO 1 mark	✓ 3,24 ✓ interval ✓ answer (3)
		[9]

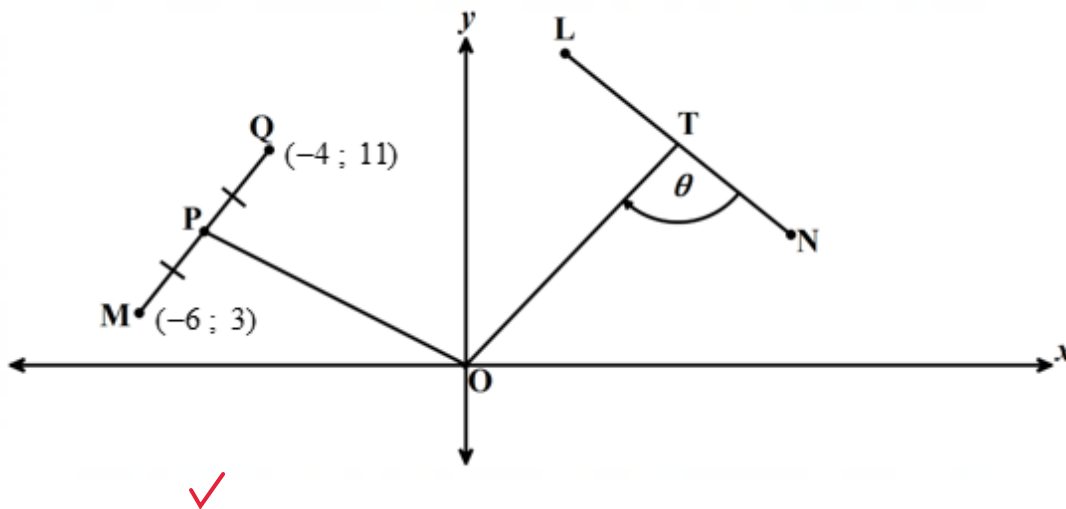
QUESTION 2

NUMBER OF HOURS WITHOUT SLEEP (x)	10	11	12	13	14	15	16	17	18	19	21	24	25
NUMBER OF MISTAKES MADE (y)	4	4	5	8	8	10	11	11	13	15	21	23	25

2.1	$25 - 10$ $= 15$	✓ Max – min ✓ answer (2)
2.2	$a = -12,05$ $b = 1,46$ $\hat{y} = -12,05 + 1,46x$	✓ $a = -12,05$ ✓ $b = 1,46$ ✓ equation (3)
2.3	$r = 0,99$	✓ answer (1)
2.4	$y = -12,05 + 1,46(23)$ $y = 21,53$ $y \approx 22$	✓ substitution ✓ answer (2)
2.5	Yes. the correlation coefficient is 0,99, <u>strong positive correlation</u> .	✓ Yes ✓ strong, positive, linear (2)
		[10]

correlation

QUESTION 3

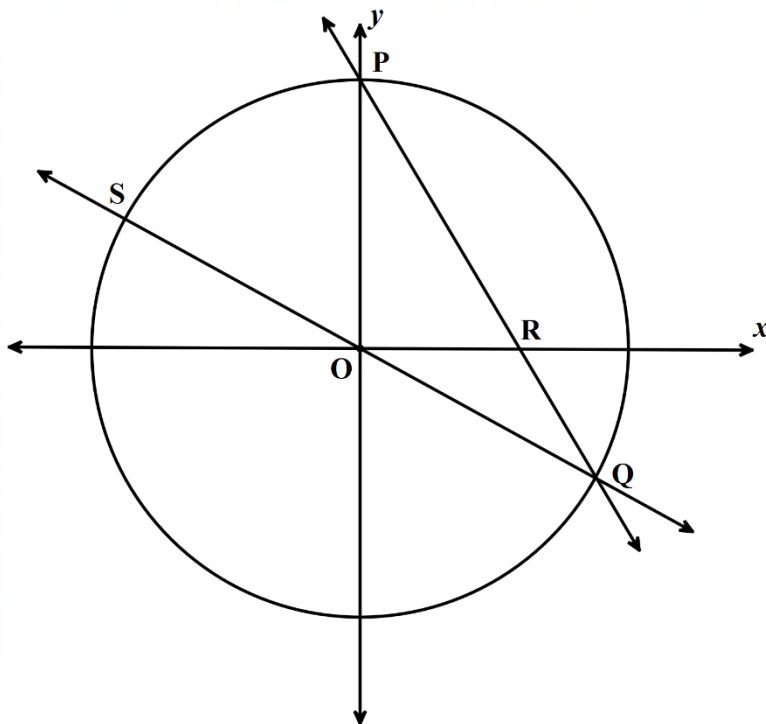


3.1.1	$\left(\frac{-6-4}{2} ; \frac{3+11}{2} \right)$ $P(-5 ; 7) \quad \checkmark \quad \checkmark$	$\checkmark x = -5$ $\checkmark y = 7$ (2)
3.1.2	$\frac{3}{2}x = 15 - x \quad \checkmark$ $3x = 30 - 2x$ $5x = 30$ $x = 6 \quad y = \frac{3}{2}(6) = 9 \quad \checkmark$ $T(6 ; 9)$ <p style="text-align: center;">OR</p> $x + y = 15$ $x + \frac{3}{2}x = 15$ $x = 6$ $y = \frac{3}{2}(6) = 9$ $T(6 ; 9)$	\checkmark equating \checkmark x-value \checkmark y-value (3) <p style="text-align: center;">OR</p> \checkmark substitution \checkmark x-value \checkmark y-value (3)

	OR	OR
	$x = 15 - y$ $y = \frac{3}{2}(15 - y)$ $y = \frac{45}{2} - \frac{3}{2}y$ $\frac{5}{2}y = \frac{45}{2}$ $y = 9$ $x + 9 = 15$ $x = 6 \quad T(6 ; 9)$	\checkmark substitution \checkmark y – value \checkmark x – value (3)
3.1.3	$y = -x + 15$ $\tan \alpha = -1$ $\alpha = 135^\circ \checkmark$ $\tan \beta = \frac{3}{2}$ $\beta = 56,31^\circ \checkmark$ $\theta = 180^\circ - 56,31^\circ - 45^\circ \checkmark$ $\theta = 78,69^\circ \checkmark$	$\checkmark \alpha = 135^\circ$ $\checkmark 56,31^\circ$ \checkmark method \checkmark answer (4)
	OR	OR
	$y = -x + 15$ $\tan \alpha = -1$ $\alpha = 135^\circ$ $\tan \beta = \frac{3}{2}$ $\beta = 56,31^\circ$ $\theta = 135^\circ - 56,31^\circ = 78,69^\circ$	$\checkmark \alpha = 135^\circ$ \checkmark method $\checkmark 56,31^\circ$ \checkmark answer (4)
3.2	$\sqrt{(-2-0)^2 + (k-1-0)^2} = 2k \checkmark$ $(-2)^2 + (k-1)^2 = 4k^2$ $4 + k^2 - 2k + 1 = 4k^2$ $3k^2 + 2k - 5 = 0 \checkmark$ $(3k + 5)(k - 1) = 0$ $k \neq -\frac{5}{3} \quad \text{or} \quad k = 1 \checkmark$	\checkmark substitution in distance formula \checkmark standard form \checkmark factors \checkmark answer <u>with</u> selection. (4)

<p>3.3.1</p>	$m_{SY} = \frac{3-5a-3}{2+4a-2} = \frac{-5a}{4a} = \frac{-5}{4} \quad \checkmark$ $m_{SU} = \frac{3-5b-3}{2+4b-2} = \frac{-5b}{4b} = \frac{-5}{4} \quad \checkmark$ $m_{YU} = \frac{3-5b-(3-5a)}{2+4b-(2+4a)} = \frac{-5b+5a}{4b-4a} = \frac{-5(b-a)}{4(b-a)} = \frac{-5}{4}$ <p>S, Y and U collinear (gradients are the same) ✓</p>	<p>✓✓ any 2 gradients</p> <p>✓ Reason</p> <p style="text-align: right;">(3)</p>
<p>3.3.2</p>	$y = \frac{-5}{4}(x) + c \quad \checkmark$ $3 = \frac{-5}{4}(2) + c \quad \checkmark$ $c = \frac{11}{2} \quad \checkmark$ $y = \frac{-5}{4}x + \frac{11}{2}$ <p style="text-align: center;">OR</p> $y - 3 = -\frac{5}{4}(x - 2)$ $y = -\frac{5}{4}x + \frac{10}{4} + 3$ $y = -\frac{5}{4}x + \frac{11}{2}$	<p>✓ $m = -\frac{5}{4}$</p> <p>✓ substitute (2 ; 3)</p> <p>✓ $c = \frac{11}{2}$</p> <p style="text-align: right;">(3)</p> <p style="text-align: center;">OR</p> <p>✓ $m = -\frac{5}{4}$</p> <p>✓ substitute (2 ; 3)</p> <p>✓ $c = \frac{11}{2}$</p> <p style="text-align: right;">(3)</p>
		<p>[19]</p>

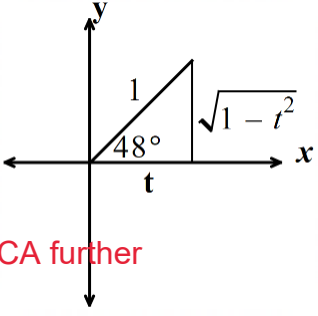
QUESTION 4



4.1	P (0 ; 4)	✓ answer (A) (1)
4.2	$y = -2x + 4$ and $x^2 + y^2 = 16$ $x^2 + (-2x + 4)^2 = 16$ $x^2 + 4x^2 - 16x + 16 - 16 = 0$ $5x^2 - 16x = 0$ $x(5x - 16) = 0$ $x \neq 0$ or $x = \frac{16}{5}$ $y = -2\left(\frac{16}{5}\right) + 4$ $y = \frac{-12}{5}$	✓ substitution ✓ standard form ✓ answer with selection ✓ substitution of x-value (4)
4.3	$y = -2x + 4$ $0 = -2x + 4$ $2x = 4$ $x = 2$ R (2 ; 0) or R(2 ; 0) Radius = 2 $(x - 2)^2 + y^2 = 4$	✓ substitute $y = 0$ ✓ $x = 2$ ✓ $(x - 2)^2 + y^2$ ✓ 4 (4)

4.4	$m_{OQ} = \frac{0 + \frac{12}{5}}{0 - \frac{16}{5}} = -\frac{3}{4} \checkmark$ <p>ref angle = $36,87^\circ$</p> <p>Inclination SQ = $180^\circ - 36,87^\circ = 143,13^\circ \checkmark$</p> <p>Inclination PR: $m = -2$, ref angle = $63,43^\circ$</p> $180^\circ - 63,43^\circ = 116,57^\circ \checkmark$ <p>Adjustment: $143,13^\circ - 116,57^\circ = 26,56^\circ \checkmark$</p>	<p>\checkmark gradient $-\frac{3}{4}$</p> <p>\checkmark $143,13^\circ$</p> <p>\checkmark $116,57^\circ$</p> <p>\checkmark $26,56^\circ$</p> <p>(4)</p>
4.5.1	$x^2 + 2x + 1 + y^2 - 6y + 9 = 6 + 1 + 9 \checkmark$ $(x+1)^2 + (y-3)^2 = 16 \checkmark$	<p>\checkmark completing the Square</p> <p>$\checkmark (x+1)^2 + (y-3)^2$</p> <p>$\checkmark 16$</p> <p>(3)</p>
4.5.2	<p>centre: $(-1 ; 3) \checkmark$ ca</p>	<p>\checkmark centre: $(-1 ; 3)$ (1)</p>
4.6	<p>Distance between 2 centres of circles: $(0 ; 0)$ and $(-1 ; 3)$</p> $\sqrt{(0+1)^2 + (0-3)^2} \checkmark \text{ ca}$ $= \sqrt{10} = 3,16 \text{ units} \checkmark$ <p>Radius + Radius = $4 + 4 = 8 \text{ units} \checkmark$</p> <p>Distance between centres < Radius₁ + Radius₂</p> <p>Circles intersect. \checkmark</p>	<p>\checkmark substitution in distance formula</p> <p>$\checkmark \sqrt{10}$</p> <p>\checkmark method</p> <p>\checkmark Conclusion</p> <p>(4)</p>
		<p>[21]</p>

QUESTION 5

5.1.1	$\cos 228^\circ$ $= -\cos 48^\circ \checkmark$ $= -t \checkmark$	$\checkmark = -\cos 48^\circ$ $\checkmark \text{ answer}$ <p style="text-align: right;">(2)</p>
5.1.2	$\sin 48^\circ = \sqrt{1 - \cos^2 48^\circ} \checkmark$ $= \sqrt{1 - t^2} \checkmark$ <p style="text-align: center;">OR</p> $\sin 48^\circ = \sqrt{1 - t^2} \checkmark \checkmark$ <div style="text-align: center;">  </div> <p style="color: red;">Penalise wrong sketch, then CA further</p>	$\checkmark \text{ identity}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(2)</p> <p style="text-align: center;">OR</p> $\checkmark \text{ sketch, Pythagoras}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(2)</p>
5.1.3	$\cos 2(48^\circ) \checkmark$ $= 2\cos^2 48^\circ - 1 \checkmark$ $= 2t^2 - 1 \checkmark$ <p style="text-align: center;">OR</p> $\cos 2(48^\circ) \checkmark$ $= 1 - 2\sin^2 48^\circ \checkmark$ $= 1 - 2(1 - t^2)$ $= 2t^2 - 1 \checkmark$ <p style="text-align: center;">OR</p> $\cos 2(48^\circ) \checkmark$ $= \cos^2 48^\circ - \sin^2 48^\circ \checkmark$ $= t^2 - (\sqrt{1 - t^2})^2$ $= 2t^2 - 1 \checkmark$	$\checkmark \text{ double angle}$ $\checkmark \text{ expansion}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p> <p style="text-align: center;">OR</p> $\checkmark \text{ double angle}$ $\checkmark \text{ expansion}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p> <p style="text-align: center;">OR</p> $\checkmark \text{ double angle}$ $\checkmark \text{ expansion}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p>

5.1.4	$\sin(45^\circ + 48^\circ) \checkmark$ $= \sin 45^\circ \cos 48^\circ + \cos 45^\circ \sin 48^\circ \checkmark$ $= \frac{\sqrt{2}}{2} \cdot t + \frac{\sqrt{2}}{2} \cdot \sqrt{1-t^2} \checkmark$	$\checkmark \sin(45^\circ + 48^\circ)$ \checkmark expansion \checkmark substitution \checkmark special angle value (4)
5.2	$\frac{\sin 36^\circ \cdot \sin \theta \cdot \cos(90^\circ - \theta)}{\sin 756^\circ} + \frac{\sin 2\theta \cdot \cos \theta}{2 \sin \theta}$ $\frac{\sin 36^\circ \cdot \sin \theta \cdot \sin \theta}{\sin 36^\circ} + \frac{2 \sin \theta \cdot \cos \theta \cdot \cos \theta}{2 \sin \theta}$ $= \sin^2 \theta + \cos^2 \theta \checkmark$ $= 1 \checkmark$ <p style="color: red; font-size: small;">if they leave out the angle - ZERO if they change another letter no penalty</p>	$\checkmark \sin \theta$ $\checkmark \sin 36^\circ$ $\checkmark 2 \sin \theta \cos \theta$ \checkmark simplify to identity. \checkmark answer (5)
5.3.1	$\tan^2 x \left(\frac{1}{\tan^2 x} - 1 \right) = \frac{\cos 2x}{\cos^2 x}$ $LHS = 1 - \tan^2 x \checkmark$ $= 1 - \frac{\sin^2 x}{\cos^2 x} \checkmark$ $= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \checkmark \checkmark$ $= \frac{\cos 2x}{\cos^2 x} = RHS$	\checkmark simplification \checkmark identity \checkmark simplification \checkmark double angle identity (4)
5.3.2	$x = 90^\circ$ $x = 0^\circ$	$x = 270^\circ$ $x = 180^\circ$; $x = 360^\circ$ \checkmark answer \checkmark answer (2)

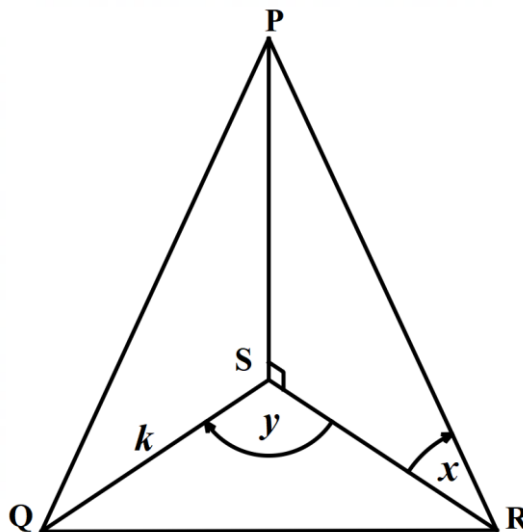
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<p>5.3.3</p>	$1 - \tan^2 x = 0$ $1 = \tan^2 x$ $\tan x = \pm 1 \quad \checkmark$ $\text{ref angle: } 45^\circ \quad \checkmark$ $x = 45^\circ + 180^\circ k, k \in \mathbb{Z}$ $x = 135^\circ + 180^\circ k, k \in \mathbb{Z} \quad \checkmark$ <p style="text-align: center;">OR</p> $\frac{\cos 2x}{\cos^2 x} = 0$ $\therefore \cos 2x = 0 \quad \checkmark$ $\text{Ref angle: } 90^\circ$ $2x = 90^\circ + 360^\circ k$ $x = 45^\circ + 180^\circ k; k \in \mathbb{Z}$ $2x = 270^\circ + 360^\circ k$ $x = 135^\circ + 180^\circ k; k \in \mathbb{Z} \quad \checkmark$	$\checkmark \tan x = \pm 1$ $\checkmark 45^\circ$ $\checkmark 135^\circ$ $\checkmark +180^\circ k; k \in \mathbb{Z}$ <p style="text-align: right;">(4)</p> <p style="text-align: center;">OR</p> $\checkmark \cos 2x = 0$ 90° $\checkmark 45^\circ$ $\checkmark 135^\circ$ $\checkmark +180^\circ k; k \in \mathbb{Z}$ <p style="text-align: right;">(4)</p>
<p>5.4</p>	$g(x) - f(x)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2 \quad \checkmark$ $\text{Maximum at } \sin x = \frac{-b}{2a} = \frac{-(-1)}{2(-2)} = -\frac{1}{4} \quad \checkmark$ $\text{ref } \angle : 14,48^\circ \quad \checkmark$ $x = 345,52^\circ \quad \checkmark$	$\checkmark -2\sin^2 x - \sin x + 2$ $\checkmark \text{substitution}$ $\checkmark \sin x = -\frac{1}{4}$ $\checkmark \text{ref angle}$ $\checkmark \text{answer}$ <p style="text-align: right;">(5)</p>
		<p>[31]</p>

QUESTION 6

<p>6.1</p>	<p> ✓ x-intercepts ✓ y-intercept ✓ Turning points $(-45^\circ; -2)$; $(135^\circ; -2)$ <i> dont have to indicate coordinates</i> </p>	<p>(3)</p>
<p>6.2</p>	<p>1 ✓</p>	<p>✓ answer (1)</p>
<p>6.3</p>	<p>$(210^\circ; -1)$ ✓ ✓</p>	<p>✓ x – value ✓ y – value (2)</p>
<p>6.4</p>	<p> $\sin 2x \cdot \cos x - \cos x = 0$ $\cos x(\sin 2x - 1) = 0$ ✓ x- intercepts: $g(x): 225^\circ$ ✓ $f(x): 90^\circ; 270^\circ$ ✓ ✓ </p>	<p> ✓ factorization ✓ 225° ✓ 90° ✓ 270° (4) </p>
		<p>[10]</p>

QUESTION 7



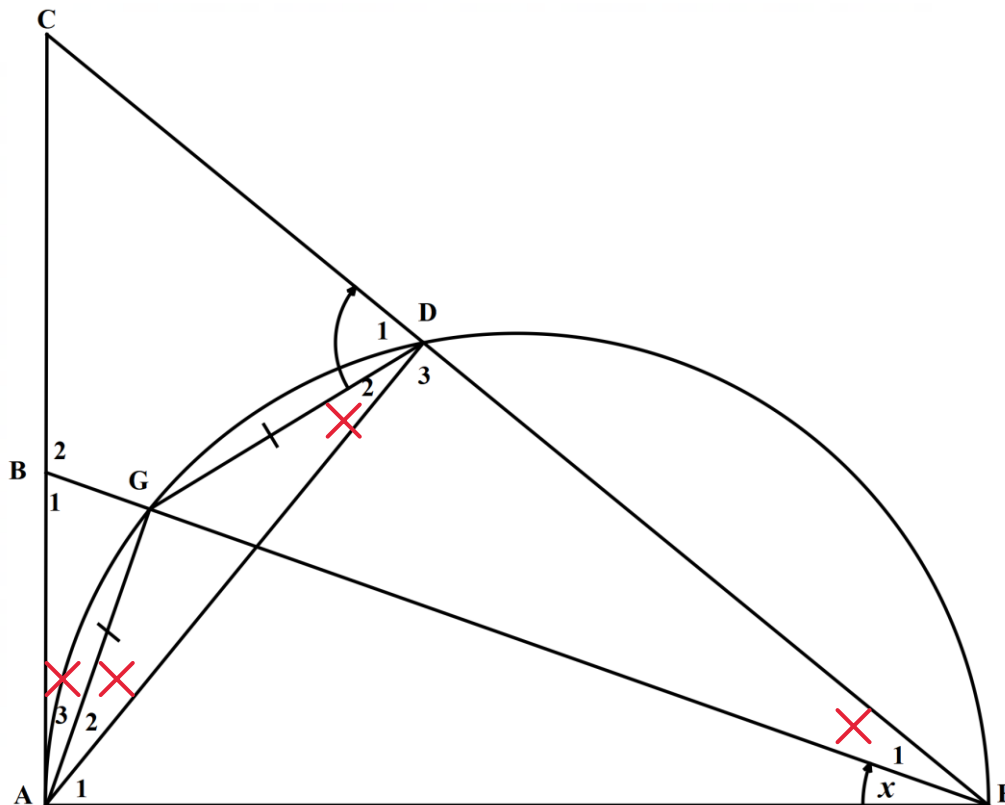
7.1	$\tan x = \frac{PS}{SR} \quad \checkmark$ $SR = \frac{PS}{\tan x} \quad \checkmark$	✓ trig ratio ✓ answer (2)
7.2	$\text{Area } \triangle QRS = \frac{1}{2} \cdot k \cdot SR \cdot \sin y \quad \checkmark$ $A = \frac{1}{2} \cdot k \cdot \left(\frac{PS}{\tan x} \right) \cdot \sin y$ $A = \frac{k \cdot PS \cdot \sin y}{2 \tan x} \quad \checkmark$ $PS = \frac{2A \tan x}{k \sin y} \quad \checkmark \quad \text{This must follow from the previous line}$	✓ Use of area rule ✓ substitute $SR = \frac{PS}{\tan x}$ ✓ simplification ✓ Change of subject (4)
7.3	$\frac{2(480,9) \cdot \tan 46,5^\circ}{87,36 \cdot \sin y} = 76,8 \quad \checkmark$ $\frac{11,6017\dots}{\sin y} = 76,8$ $\frac{11,6017\dots}{76,8} = \sin y$ $0,151\dots = \sin y \quad \checkmark$ $y = 8,69^\circ \quad \checkmark$	✓ substitution ✓ simplification ✓ answer (3)
		[9]

QUESTION 8

8.1	$\hat{E} = 60^\circ$ (ext \angle of cyclic quad / buite \angle kdvh)	✓S ✓R (2)
8.2	$\hat{O}_1 = 120^\circ$ (\angle at centre = 2 \times \angle at circumference /Middelpuntshoek = 2 \times omtrekshoek)	✓S ✓R (2)
8.3	$\hat{D}_1 = \frac{180^\circ - 120^\circ}{2} = 30^\circ$ (Sum of int \angle 's Δ , \angle 's opp equal sides) ✓ (Som v binne \angle^e v Δ , hoeke teenoor gelyke sye)	✓S ✓R (2)
		[6]

Gelykbenige driehoek word nie aanvaar as 'n rede nie.
 Isc traingle is not accepted as a reason

QUESTION 9

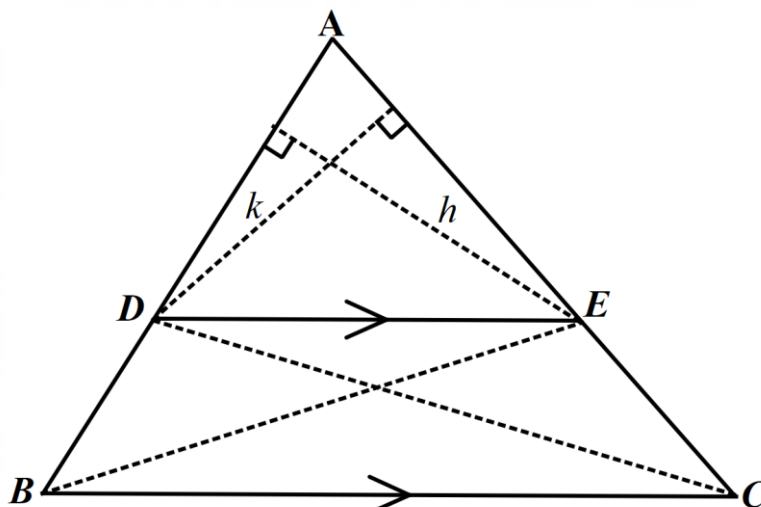


9.1	<p>ANY THREE OF THE FOUR ANGLES WITH REASONS IN CORRECT ORDER</p> <p>$\hat{A}_3 = x$ (tan-chord th / raaklyn-koord st) ✓ s/r</p> <p>$\hat{D}_2 = x$ (\angle's in the same segment/ hoeke in dieselfde segment) ✓</p> <p>OR (tan-chord th / raaklyn-koord st)</p> <p>$\hat{A}_2 = x$ (angles opp = sides / hoeke teenoor gelyke sye) ✓</p> <p style="text-align: center;">OR</p> <p>$\hat{A}_3 = x$ (tan-chord th / raaklyn-koord st) ✓ s/r</p> <p>$\hat{D}_2 = x$ (\angle's in the same segment/ hoeke in dieselfde segment) ✓</p> <p>OR (tan-chord th / raaklyn-koord st)</p> <p>$\hat{E}_1 = x$ (\angle's subtended by = chords / Hoeke onderspan deur gelyke koorde) ✓</p> <p style="text-align: center;">OR</p> <p>$\hat{A}_3 = x$ (tan-chord th / raaklyn-koord st) ✓ s/r</p> <p>$\hat{E}_1 = x$ (\angle's subtended by = chords / Hoeke onderspan deur gelyke koorde) ✓</p> <p>$\hat{A}_2 = x$ (angles opp = sides / hoeke teenoor gelyke sye) ✓</p>	<p>✓S/R</p> <p>✓S ✓R</p> <p>✓S ✓R</p> <p>(5)</p> <p style="text-align: center;">OR</p> <p>✓S/R</p> <p>✓S ✓R</p> <p>✓S ✓R</p> <p>(5)</p> <p style="text-align: center;">OR</p> <p>✓S/R</p> <p>✓S ✓R</p> <p>✓S ✓R</p> <p>(5)</p>
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You can ALSO get three angles WITHOUT angle A3. Then award 5 marks.

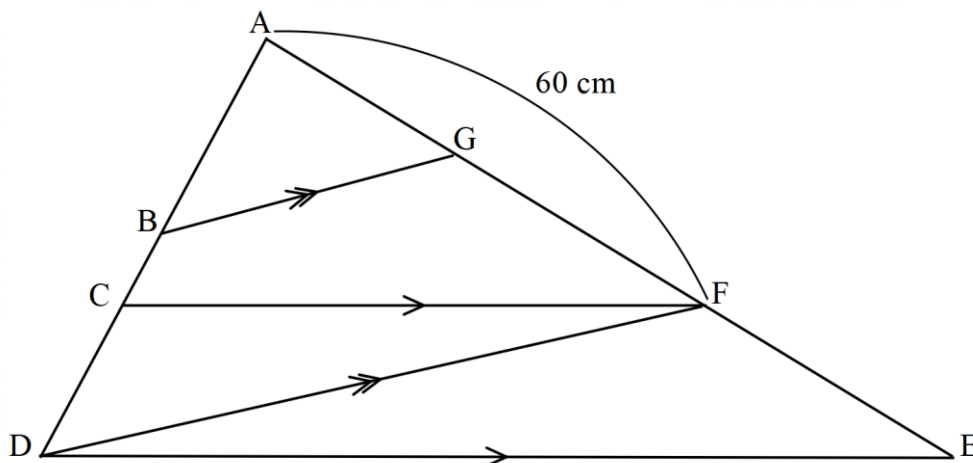
9.2	<p>$\hat{D}_3 = 90^\circ$ (\angle's in a semi-circle / Hoeke in 'n halwe sirkel) ✓ s/r</p> <p>$\hat{D}_1 = 180^\circ - 90^\circ - x$ $= 90^\circ - x$ ✓ s (Angles on a straight line/ Hoeke op 'n reguit lyn)</p> <p>$\hat{A} = 90^\circ$ (Rad \perp Tangent / Rad \perp Raaklyn) ✓ s/r</p> <p>$\hat{B}_2 = 90^\circ + x$ (Ext angle of Δ / Buite hoek v Δ) ✓ s</p> <p>$\therefore \hat{B}_2 + \hat{D}_1 = 180^\circ$</p> <p>BCDG a cyclic quad (Opp angles of <u>quad</u> supp) ✓ R</p> <p>BCDG 'n Koordevierhoek (Teenoorst hoeke van vierhoek supp) OR CONVERSE opp angles cyclic quad</p> <p style="text-align: center;">OR</p> <p>$\hat{D}_3 = 90^\circ$ (\angle's in a semi-circle / Hoeke in 'n halwe sirkel) ✓ s/r</p> <p>$\hat{D}_2 + \hat{D}_3 = 90^\circ + x$ ✓ s</p> <p>$\hat{A} = 90^\circ$ (Rad \perp Tangent / Rad \perp Raaklyn) ✓ s/r</p> <p>$\hat{B}_2 = 90^\circ + x$ (Ext angle of Δ / Buite hoek v Δ) ✓ s</p> <p>$\hat{D}_2 + \hat{D}_3 = 90^\circ + x = \hat{B}_2$</p> <p>BCDG a cyclic quad (Ext angle of quad = opp int angle) ✓ R</p> <p>BCDG 'n Koordevierhoek (Buite hoek van vierhoek = teenoorstaande binnehoek)</p>	<p>✓S/R</p> <p>✓S $90^\circ - x$</p> <p>✓S/R</p> <p>✓S $90^\circ + x$</p> <p>✓R (5)</p> <p style="text-align: center;">OR</p> <p>✓S/R</p> <p>✓S $\hat{D}_2 + \hat{D}_3 = 90^\circ + x$</p> <p>✓SR</p> <p>✓S $\hat{B}_2 = 90^\circ + x$</p> <p>✓R (5)</p> <p style="text-align: right;">[10]</p>
or (converse Ext angle cyclic quad)		

QUESTION 10



Construction need only be on sketch or in words. No construction - no further marking

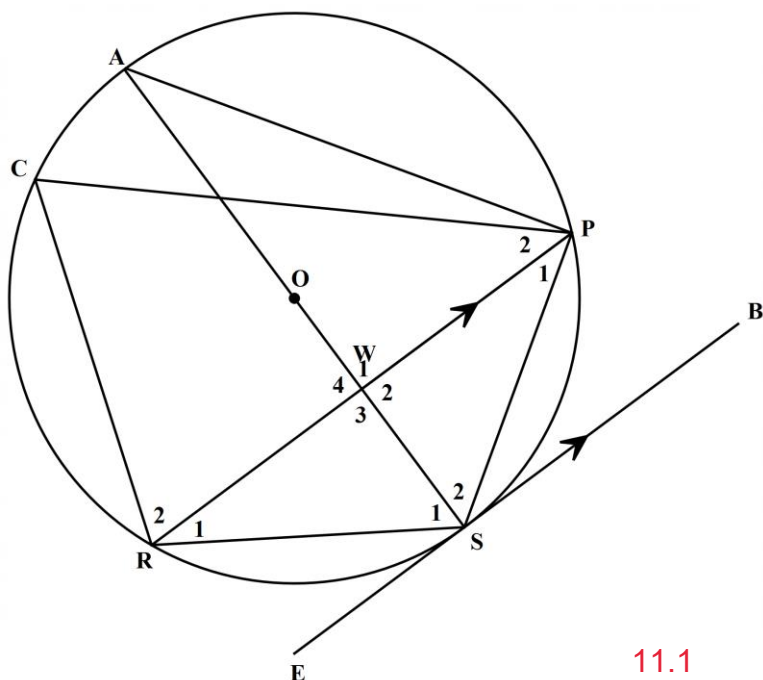
10.1	Constr: Draw heigh h with base AD and height k with base AE ✓ Draw DC and BE	✓construction
	PROOF: $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \cdot AD \cdot h}{\frac{1}{2} \cdot BD \cdot h} = \frac{AD}{BD}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle CDE} = \frac{\frac{1}{2} \cdot AE \cdot k}{\frac{1}{2} \cdot EC \cdot k} = \frac{AE}{EC}$	✓S ✓S
	Area $\triangle BDE$ = Area $\triangle CDE$ (Same base, same height/ Dieselfde basis en hoogte)	✓S /R
	$\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle CDE} \quad \checkmark$	✓S
	$\frac{AD}{BD} = \frac{AE}{EC}$	(5)



<p>10.2.1</p>	<p>$60 \div 3 \times 2 = 40\text{cm}$ ✓ ✓</p> <p style="text-align: center;">OR</p> <p>$\frac{FE}{60} = \frac{2}{3}$ $\therefore FE = 40\text{cm}$ ✓ ✓</p> <p style="text-align: center;">ANSWER ONLY – FULL MARKS</p>	<p>✓ method ✓ answer (2)</p> <p style="text-align: center;">OR</p> <p>✓ S ✓ answer (2)</p>
<p>10.2.2</p>	<p>$\frac{AF}{FE} = \frac{AC}{CD} = \frac{3}{2}$ (Line parallel one side of Δ / Lyn parallel een sy van Δ) ✓ s/r</p> <p>$\frac{AG}{GF} = \frac{AB}{BD} = \frac{7}{8}$ (Line parallel one side of Δ / Lyn parallel een sy van Δ) ✓ s/r</p> <p style="text-align: center;"> $\begin{array}{ccccccc} & 7x & & 8x & & & \\ & \bullet & \text{---} & \bullet & \text{---} & \bullet & \text{---} & \bullet \\ A & & B & C & & D & & \end{array}$ $\begin{array}{ccc} & 3y & & 2y & \\ & \bullet & \text{---} & \bullet & \end{array}$ </p> <p>From ratios: $AB = 7x$ $BD = 8x$ $AC = 3y$ $CD = 2y$</p> <p>$5y = 15x$ $y = 3x$ ✓ method MUST HAVE</p> <p>$\frac{BC}{CD} = \frac{8x - 2y}{2y}$</p> <p>$\frac{BC}{CD} = \frac{8x - 2(3x)}{2(3x)} = \frac{2x}{6x} = \frac{1}{3}$ ✓</p>	<p>✓ S/R ✓ S/R</p> <p>✓ method</p> <p>✓ Answer (4)</p>

	<p style="text-align: center;">OR</p> <p>$DC = \frac{2}{5} AD$ (prop th, CF//DE / eweredigheids st, CF//DE) ✓ s/r</p> <p>$BD = \frac{8}{15} AD$ (prop th, BG//DF / eweredigheidst, BG//DF) ✓ s/r</p> <p>$BC = BD - DC$</p> <p>$BC = \frac{8}{15} AD - \frac{2}{5} AD = \frac{2}{15} AD$ ✓ method</p> <p>$\frac{BC}{CD} = \frac{\frac{2}{15} AD}{\frac{2}{5} AD} = \frac{1}{3}$ ✓</p>	<p style="text-align: center;">OR</p> <p>✓ SR</p> <p>✓ SR</p> <p>✓ method</p> <p>✓ Answer</p> <p style="text-align: right;">(4)</p>
		[11]

QUESTION 11



11.1
Penalise for // left out

<p>11.1</p>	<p>$\hat{O}S\hat{B} = 90^\circ$ (Rad \perp tangent / Rad \perp raaklyn) ✓ s $\hat{W}_1 = 90^\circ$ (Corr \angle's PR//ED / ooreenk \angle^e PR//ED) ✓ s/r WR=WP (Line from centre \perp to chord / lyn uit mdpt \perp op koord) ✓ r</p>	<p>✓S✓R ✓S/R ✓R (4)</p>
<p>11.2</p>	<p>In ΔWRS and ΔPAS: $\hat{A}P\hat{S} = 90^\circ$ (\angle in a semi circle / Hoek in semi sirkel) ✓ s/r $\hat{W}_3 = 90^\circ$ (Vertically opp \angle's / Regoorst hoeke) ✓ s/r $\hat{W}_3 = \hat{A}P\hat{S}$ $\hat{A} = \hat{R}$ (\angle's in the same segment / \angle^e in dieselfde segment) ✓ s/r $\Delta PAS \equiv \Delta WRS$ (AAA) ✓ r</p>	<p>✓S/R ✓ s/r ✓S/R ANY 2 ANGL ES ✓R (4)</p>
<p>11.3</p>	<p>ΔWRS and ΔWPS: $\hat{W}_3 = \hat{W}_2 = 90^\circ$ (\angle's on a straight line / Hoeke op 'n reguit lyn) ✓ s/r WR = WP (Proven / Bewys) ✓ s WS is common/gemeenskaplik ✓ s $\Delta WRS \equiv \Delta WPS$ (SAS) ✓ order and reason $\therefore \Delta PAS \equiv \Delta WPS$ $\frac{AS}{PS} = \frac{PS}{WS}$ (From $\equiv \Delta$) ✓ r $PS^2 = AS \cdot WS$</p>	<p>✓S/R ✓S ✓S ✓Order and Reason ✓S ✓R (6)</p>

	OR/OF ✓ s/r	OR/OF
	$\frac{WS}{RS} = \frac{PS}{AS}$ (corresponding sides in proportion, ooreenkomstige sye in verhouding)	✓ S/R
	$RS \cdot PS = WS \cdot AS$ ✓ s	✓ S
	ΔWRS and ΔWPS ✓ s $\hat{W}_3 = \hat{W}_2 = 90^\circ$ (\angle 's on a straight line / Hoeke op 'n reguit lyn)	✓ S
	$WR = WP$ (Proven / Bewys) ✓ s	✓ S
	WS is common/gemeenskaplik	✓ Order
	$\Delta WRS \equiv \Delta WPS$ (SAS) ✓ order and reason	and Reason
	$\Rightarrow RS = PS$ ✓ s	✓ S
	$PS^2 = AS \cdot WS$	(6)
		[14]

TOTAL: 150

**ALTERNATIVE:
 TRIANGLE APS /// TRIANGLE PWS**



GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION
VOORBEREIDENDE EKSAMEN

2023

MARKING
GUIDELINES/NASIENRIGLYNE

MATHEMATICS/*WISKUNDE* (PAPER/*VRAESTEL 2*) (10612)

19 pages/*bladsye*

NOTE:

- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in **ALL** aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is **NOT** acceptable.

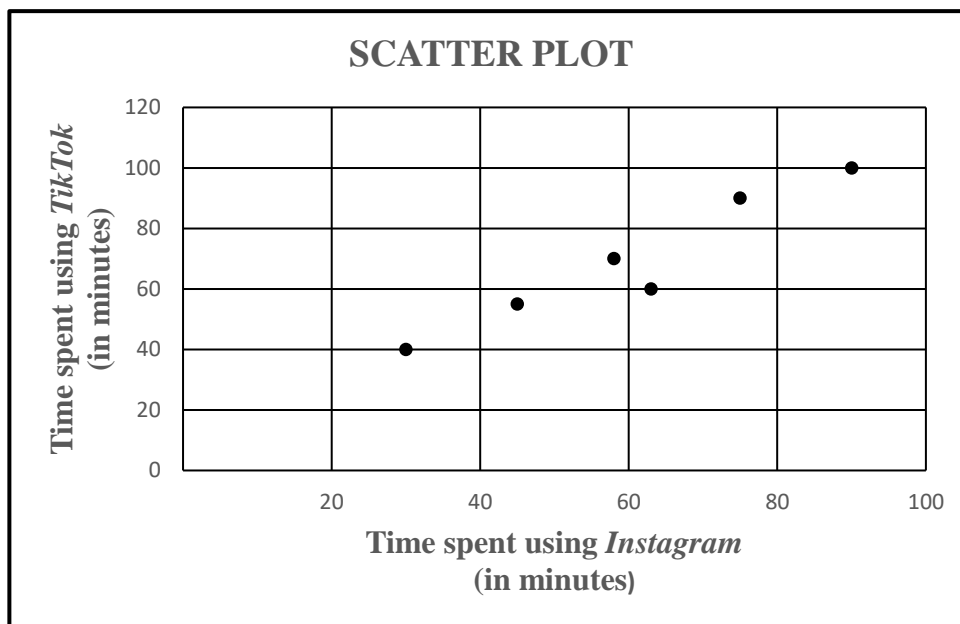
LET WEL:

- As 'n kandidaat 'n vraag **TWEE KEER** beantwoord, sien slegs die **EERSTE** poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aannames van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY/MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede.)</i>
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is.)</i>
S/R	Award a mark if the statement AND reason are both correct.
	<i>(Ken 'n punt toe as beide die bewering EN rede korrek is.)</i>

QUESTION/VRAAG 1

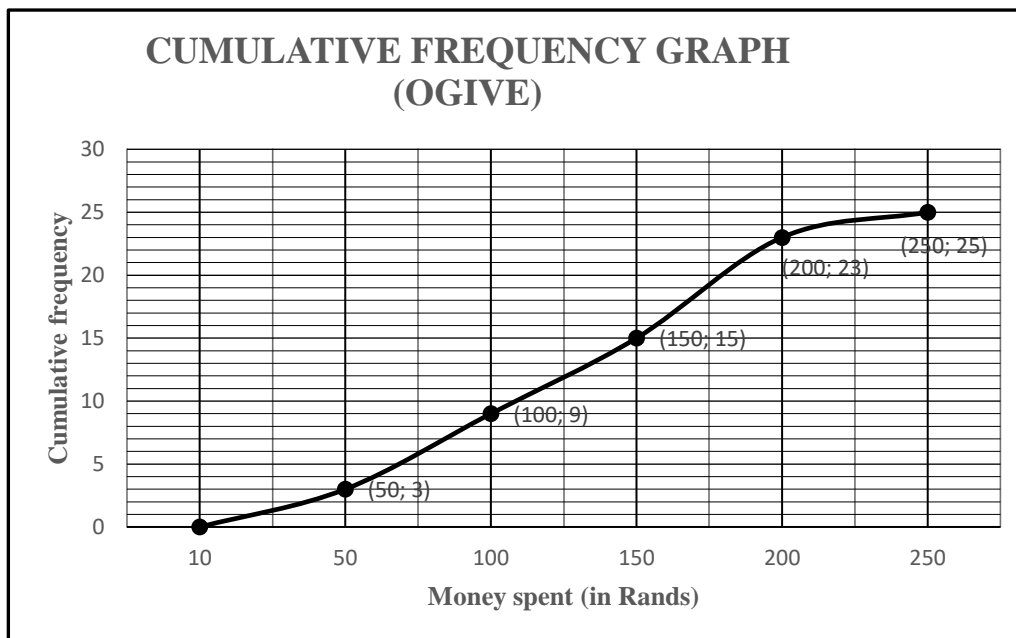
TIME USED ON INSTAGRAM (in minutes)	30	45	58	63	75	90
TIME USED ON TIKTOK (in minutes)	40	55	70	60	90	100



1.1	$r = 0,96$	✓ answer/antwoord (1)
1.2	Very strong /Baie sterk	✓ answer/antwoord Accept: Strong Aanvaar: Sterk (1)
1.3	$a = 8,12$ $b = 1,01$ $\hat{y} = 8,12 + 1,01x$	✓ $a = 8,12$ ✓ $b = 1,01$ ✓ $\hat{y} = 8,12 + 1,01x$ <div style="border: 1px solid black; padding: 5px; text-align: center;"> Answer only: Full marks Slegs antwoord: Volpunte </div> (3)
1.4	$\hat{y} = 8,12 + 1,01(115)$ $= 124,27 \approx 124$ minutes/minute OR $\hat{y} = 124,80 \approx 125$ minutes / minute (calculator)	✓ substitute/vervang 115 ✓ answer/antwoord OR ✓ ✓ answer/antwoord (2)

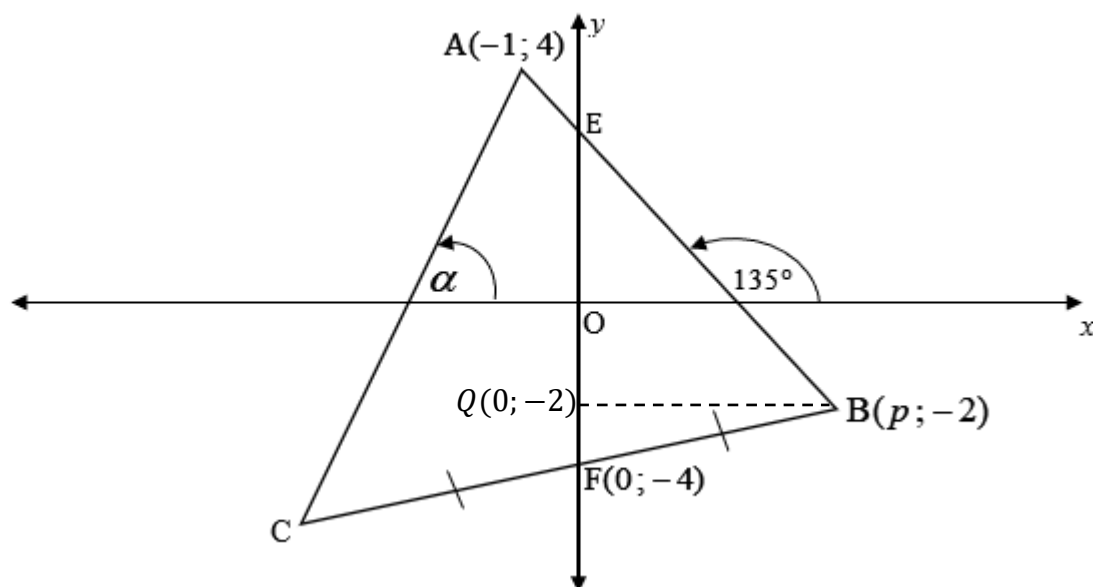
1.5	$73,4 = \frac{\sum x}{10} \quad \mathbf{OR/OF} \quad 73,4 = \frac{\sum x}{20}$ <p>Total/Totaal = 734 minutes/minute</p> <p>Total for Instagram and TikTok = 1 468 minutes Totaal vir Instagram en TikTok = 1 468 minute</p> $\frac{1\ 468}{60} = 24,47$ <p>∴ Yes I agree. More than a day was spent./ ∴ Ja, ek stem saam. Meer as 'n dag is spandeer.</p>	<p>✓ 1 468 minutes/minute</p> $\checkmark \frac{1468}{60} = 24,47 \text{ hours/ure}$ <p>✓ conclusion/gevolgtrekking</p> <p>(3)</p> <p>[10]</p>
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QUESTION/VRAAG 2



2.1.1	25 learners/ <i>leerders</i>	✓ answer/ <i>Antwoord</i> (1)
2.1.2	$a = 3$ $b = 6$	✓ $a = 3$ ✓ $b = 6$ (2)
2.1.3	$25 - 19 = 6$ learners/ <i>leerders</i> $\frac{6}{25} \times 100 = 24\%$ $\therefore 24\%$ of the learners spent more than R175/ <i>van die leerders het meer as R175 gespandeer</i>	✓ 6 ✓ 24% (2)
2.2	$\bar{x} - 2\sigma = 4,8$ $13,20 - 2\sigma = 4,8$ $-2\sigma = -8,4$ $\sigma = 4,2$ OR $\bar{x} + 2\sigma = 9,2$ $13,20 + 2\sigma = 9,2$ $2\sigma = -4$ OR The interval is incorrect/meaningless because $13,20 < 9,2$ which is mean < upperlimit, therefore no further calculations can be done. / <i>Die gegewe interval is verkeerd / betekenisloos, want $13,20 < 9,2$, dit is die gemiddeld < boonste limiet. Dus kan geen verdere berekenings gemaak word nie.</i>	✓ ✓ equation/ <i>vergelyking</i> ✓ ✓ $\sigma = 4,2$ OR ✓ ✓ equation/ <i>vergelyking</i> ✓ ✓ $2\sigma = -4$ OR ✓ ✓ ✓ ✓ explanation/ <i>verduideliking</i> (4)
		[9]

QUESTION/VRAAG 3

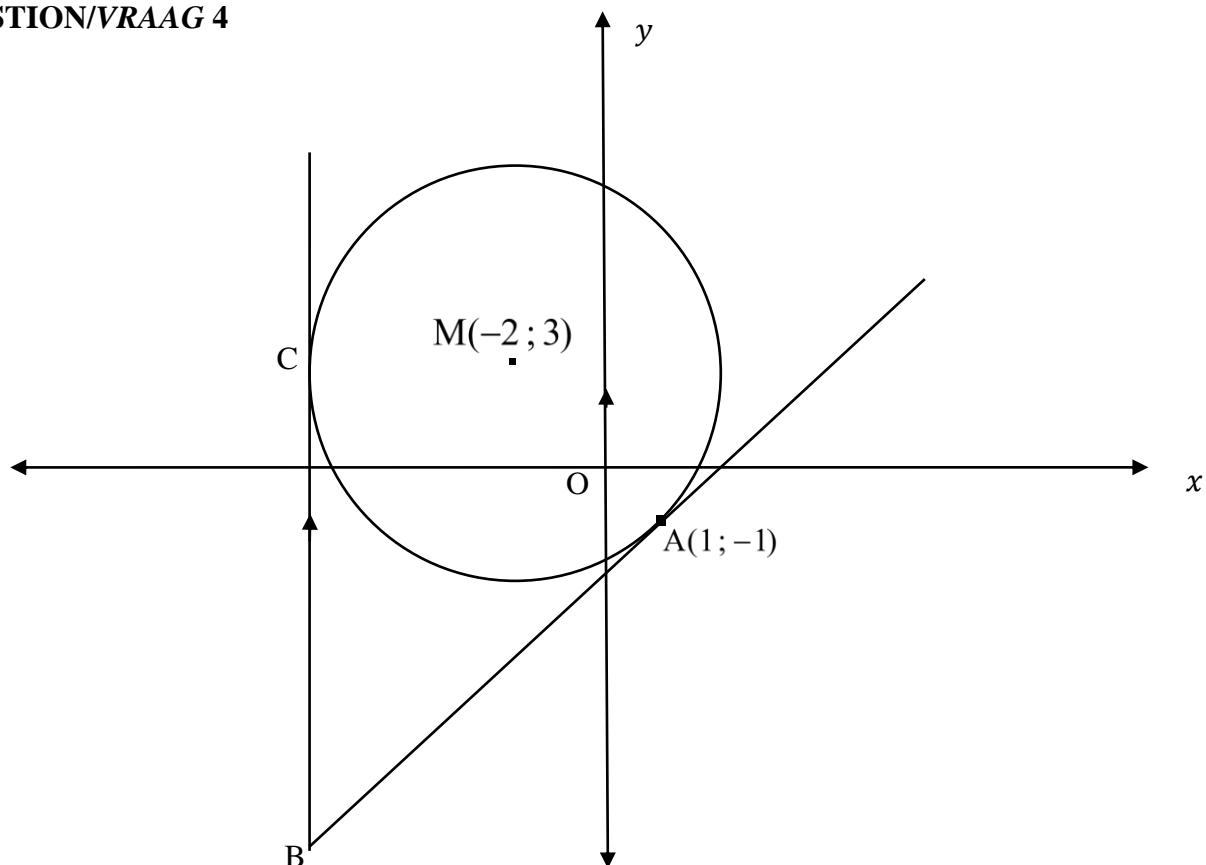


3.1	$m_{AB} = \tan 135^\circ = -1$	✓ $m_{AB} = \tan 135^\circ$ ✓ answer (2)
3.2	$m_{AB} = \frac{-2-4}{p-(-1)}$ $-1 = \frac{-6}{p+1}$ $p+1 = 6$ $p = 5$	✓ correct substitution of A and B into gradient formula/korrekte substitusie van A en B in die gradiënt formule ✓ equate to/gelykstel aan -1 (2)
3.3	$\frac{5+x_c}{2} = 0 \quad \text{and/en} \quad \frac{-2+y_c}{2} = -4$ $x_c = -5 \quad \text{and/en} \quad y_c = -8+2$ $C(-5; -6) \quad y_c = -6$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>NOTE: Does not have to be in coordinate form <i>Let op: Dit is nie nodig om in koördinaatvorm te skryf nie.</i></p> </div>	✓ $x_c = -5$ ✓ $y_c = -6$ (2)

3.4	$m_{AC} = \frac{4 - (-6)}{-1 - (-5)} = \frac{10}{4} = \frac{5}{2}$ <p>Substitute/Vervang A(-1 ; 4)</p> $y - 4 = \frac{5}{2}(x - (-1)) \quad \text{OR/OF} \quad 4 = \frac{5}{2}(-1) + c$ $y = \frac{5}{2}x + \frac{5}{2} + 4 \quad 4 = \frac{-5}{2} + c$ $y = \frac{5}{2}x + \frac{13}{2} \quad c = 6\frac{1}{2}$ $y = \frac{5}{2}x + 6,5 \quad y = \frac{5}{2}x + 6\frac{1}{2}$	<p>✓ correct substitution of A and C into gradient formula/korrekte substitusie van A en C in die gradiënt formule</p> <p>✓ $m_{AC} = \frac{5}{2}$</p> <p>✓ subs of m and point A or C/vervang m en punt A of C</p> <p>✓ equation/vergelyking (4)</p>
3.5	$\tan \alpha = \frac{5}{2}$ $\alpha = 68,20^\circ$ $\hat{CAB} = 135^\circ - 68,20^\circ \quad (\text{Ext } \angle \text{ of } \Delta)$ $= 66,8^\circ$	<p>✓ $\tan \alpha = \frac{5}{2}$</p> <p>✓ $\alpha = 68,20^\circ$</p> <p>✓ answer/antwoord (3)</p>
3.6	<p>Subst./Vervang A (-1 ; 4) OF/OF Subst/Vervang B(5; -2)</p> $E : 4 = -1(-1) + c \quad \text{OR/OF} \quad y + 2 = -1(x - 5)$ $3 = c \quad y + 2 = -x + 5$ $y = -x + 3$ <p>E(0 ; 3) F(0 ; -4)</p> $\text{Area/Opp } \triangle BEF = \frac{1}{2} \times EF \times \perp h$ $= \frac{1}{2} \times 7 \times 5$ $= 17,5 \text{ square units/vierkante eenhede}$ <p>OR/OF</p> $E : 4 = -1(-1) + c$ $3 = c$ <p>E(0 ; 3) $\hat{FEB} = 45^\circ$</p> $BE^2 = (5 - 0)^2 + (-2 - 3)^2$ $BE = \sqrt{50}$ $\text{Area/Opp } \triangle BEF = \frac{1}{2} \times EF \times BE \times \sin \hat{FEB}$ $= \frac{1}{2} \times 7 \times \sqrt{50} \times \sin 45^\circ$ $= 17,5 \text{ square units / vierkante eenhede}$	<p>✓ value of c/waarde van c</p> <p>✓ \perp height/hoogte = 5</p> <p>✓ answer/antwoord (3)</p> <p>OR/OF</p> <p>✓ value of c/waarde van c</p> <p>✓ $BE = \sqrt{50}$</p> <p>✓ answer/antwoord (3)</p>

	<p>OR/OF $E : 4 = -1(-1) + c$ $3 = c$ $E(0 ; 3)$</p> <p>$Area / Opp \Delta FQB = \frac{1}{2} \times 2 \times 5 = 5$</p> <p>$Area / Opp \Delta EQB = \frac{1}{2} \times 5 \times 5 = 12,5$</p> <p>$Area / Opp \Delta EFB = 5 + 12,5$ $= 17,5 \text{ square units / vierkante eenhede}$</p>	<p>OR/OF</p> <p>✓ value of c/waarde van c</p> <p>✓ \perp height/hoogte = 5</p> <p>✓ answer/antwoord (3)</p>
3.7	<p>$AK = 5\sqrt{5}$</p> <p>$\sqrt{(t+1)^2 + (t-4)^2} = (5\sqrt{5})$</p> <p>$t^2 + 2t + 1 + t^2 - 8t + 16 = 125$</p> <p>$t^2 - 3t - 54 = 0$</p> <p>$(t-9)(t+6) = 0$</p> <p>$t = 9 \text{ (n.a) or/of } t = -6$</p> <p>$K(-6 ; -6)$</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>NOTE: Must be in coordinate form. <i>Let op: Moet in koördinaatvorm wees.</i></p> </div>	<p>✓ substitute A and K into distance formula/ vervang A en K in die afstandformule</p> <p>✓ equating/gelykstelling</p> <p>✓ standard form/ standaardvorm</p> <p>✓ factors/faktore</p> <p>✓ $K(-6 ; -6)$ (5)</p>
		[21]

QUESTION/VRAAG 4

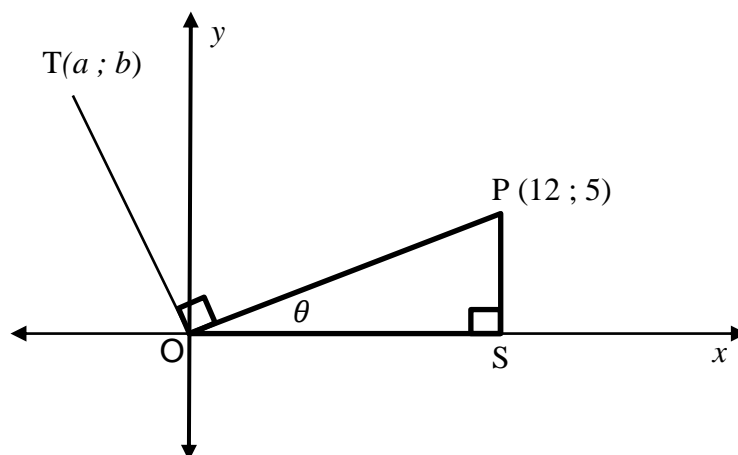


4.1	$(x+2)^2 + (y-3)^2 = r^2$ $(1+2)^2 + (-1-3)^2 = r^2$ $9+16 = r^2$ $\therefore (x+2)^2 + (y-3)^2 = 25$ <p>OR/OF</p> $AM = \sqrt{(-2-1)^2 + (3+1)^2}$ $AM = \sqrt{9+16}$ $r = 5$ $\therefore r^2 = 25$ $\therefore (x+2)^2 + (y-3)^2 = 25$	<p>✓ subs of M and A into the distance formula/ vervang M en A in die afstandformule</p> <p>✓ $r^2 = 25$</p> <p>✓ equation/vergelyking</p> <p>OR/OF</p> <p>✓ subs of M and A into the distance formula/ vervang M en A in die afstandformule</p> <p>✓ $r^2 = 25$</p> <p>✓ equation/vergelyking (3)</p>
4.2	<p>C(-2-5; 3) (by symmetry/deur simmetrie)</p> <p>C(-7; 3)</p>	<p>✓ $x_C = -7$</p> <p>✓ $y_C = 3$ (2)</p>

4.3	$m_{AM} = \frac{3 - (-1)}{-2 - 1} = -\frac{4}{3}$ $m_{AB} = \frac{3}{4} \quad (\text{radius} \perp \text{tangent/raaklyn})$ $y - (-1) = \frac{3}{4}(x - 1) \quad (\text{subst./vervang } A(1; -1))$ $y = \frac{3}{4}x - \frac{3}{4} - 1$ $y = \frac{3}{4}x - \frac{7}{4}$	<p>✓ subs A and M into gradient formula/ vervang A en M in die gradiënt formule</p> <p>✓ $m_{\text{radius}} = -\frac{4}{3}$</p> <p>✓ $m_{\text{tangent}} = \frac{3}{4}$</p> <p>✓ substitute m and A/vervang m en A</p> <p>✓ equation/vergelyking (5)</p>
4.4	<p>B(-7; y_B)</p> <p>subs./vervang B(-7; y_B) into eqn of AB/in die vergelyking van AB</p> $y_B = \frac{3}{4}(-7) - \frac{7}{4}$ $y_B = -7$ <p>B(-7; -7)</p> <p>BC = 10 units/eenhede</p>	<p>✓ subs./vervang $x = -7$</p> <p>✓ $y_B = -7$</p> <p>✓ BC = 10 (3)</p>
4.5	$(x - 1)^2 + (y + 1)^2 = 1$	<p>✓ LHS/LK</p> <p>✓ RHS/RK (2)</p>
4.6	$r_2 - r_1 < MN < r_2 + r_1$ $1 < p + 2 < 9 \text{ or/of } 1 < -2 - p < 9$ $-1 < p < 7 \text{ or/of } 3 < -p < 11$ $-1 < p < 7 \text{ or/of } -11 < p < -3$ <p>OR/OF</p> <p>Two circles will touch when:/ Twee sirkels sal sny indien:</p> <p>Distance between centre/Afstand tussen middelpunte = $r_1 + r_2$ or /of</p> <p>distance between centres/ Afstand tussen middelpunte = $r_2 - r_1$</p> $\sqrt{(p+2)^2 + (3-3)^2} = 4+5 \text{ or/of } \sqrt{(p+2)^2 + (3-3)^2} = 5-4$ $(p+2)^2 = 9^2 \text{ or/of } (p+2)^2 = 1$ $p+2 = \pm 9 \text{ or/of } p+2 = \pm 1$ $p = \pm 9 - 2 \text{ or/of } p = -2 \pm 1$ $p = -11 \text{ or/of } p = 7 \text{ or/of } p = -3 \text{ or/of } p = -1$ <p>Two circles intersect in two different points if / twee sirkels sny in twee verskillende punte as</p> $-11 < p < -3 \text{ or } -1 < p < 7$	<p>✓ value of 1 and 9 (Sum and difference of radii)/Waarde van 1 en 9 (som en verskil van radiusse)</p> <p>✓✓ $-1 < p < 7$</p> <p>✓✓ $-11 < p < -3$ (5)</p> <p>OR/OF</p> <p>✓ value of 1 and 9 (Sum and difference of radii)/Waarde van 1 en 9 (som en verskil van radiusse)</p> <p>✓✓ $-1 < p < 7$</p> <p>✓✓ $-11 < p < -3$ (5)</p>
		[20]

QUESTION/VRAAG 5

5.1



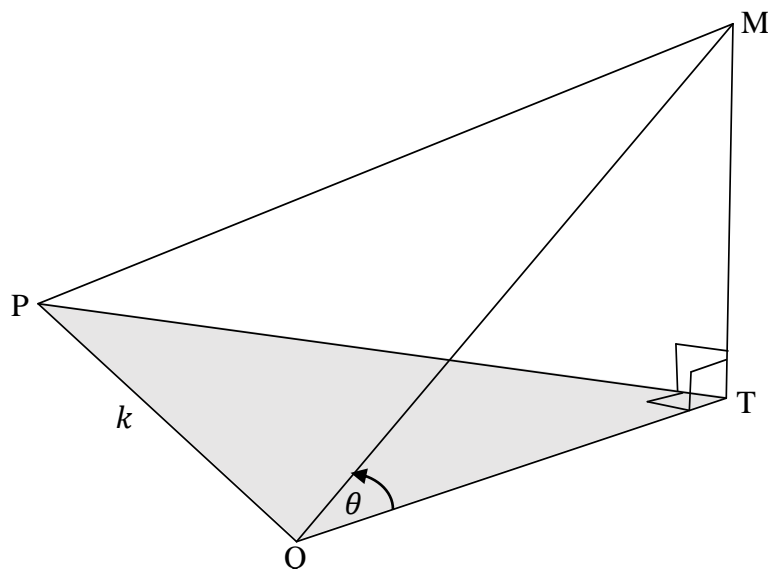
5.1.1	$\tan \theta = \frac{5}{12}$	✓ answer/antwoord (1)
5.1.2	$r^2 = 5^2 + 12^2$ $r = 13$ $\sin \theta = \frac{5}{13}$	✓ value of/waarde van r ✓ answer/antwoord <div style="border: 1px solid black; padding: 5px; display: inline-block;"> Answer only: Full marks Slegs antwoord: Volpunte </div> (2)
5.1.3	$\cos(90^\circ + \theta) = \frac{a}{19,5}$ $-\sin \theta = \frac{a}{19,5}$ $-\frac{5}{13} = \frac{a}{19,5}$ $a = \frac{-5 \times 19,5}{13}$ $a = -7,5$	✓ ratio/verhouding ✓ $-\sin \theta$ ✓ substitute/vervang $\sin \theta$ ✓ value of/waarde van a (4)
5.2	$\frac{\sin(360^\circ - 2x) \cdot \sin(-x)}{\sin(90^\circ + x)} + 2 \cos^2(180^\circ + x)$ $= \frac{(-\sin 2x) \cdot (-\sin x)}{\cos x} + 2 \cos^2 x$ $= \frac{(-2 \sin x \cdot \cos x) \cdot (-\sin x)}{\cos x} + 2 \cos^2 x$ $= 2 \sin^2 x + 2 \cos^2 x$ $= 2(\sin^2 x + \cos^2 x)$ $= 2$	✓ $-\sin 2x$ ✓ $-\sin x$ ✓ $\cos x$ ✓ $2 \cos^2 x$ ✓ $\sin 2x = 2 \sin x \cdot \cos x$ ✓ answer/antwoord (6)

5.3	$\cos 42^\circ = 2 \cos^2 21^\circ - 1$ $\sqrt{k} = 2 \sin^2 69^\circ - 1$ $\frac{\sqrt{k} + 1}{2} = \sin^2 69^\circ$	✓ double angle expansion/ <i>dubbelhoek uitbreiding</i> ✓ co-function/ <i>ko-funksie</i> ✓ answer/ <i>antwoord</i> (3)
5.4.1	$\begin{aligned} \text{LHS/LK} &= \frac{\sin 5x \cdot \cos 3x - \cos 5x \cdot \sin 3x}{\tan 2x} - 1 \\ &= \frac{\sin(5x - 3x)}{\frac{\sin 2x}{\cos 2x}} - 1 \\ &= \cos 2x - 1 \\ &= (1 - 2 \sin^2 x) - 1 \\ &= -2 \sin^2 x \\ &= \text{RHS} \end{aligned}$	✓ $\sin(5x - 3x)$ ✓ $\tan 2x = \frac{\sin 2x}{\cos 2x}$ ✓ $\cos 2x - 1$ ✓ double angle <i>identity/dubbelhoek identiteit</i> (4)
5.4.2	Undefined if/ <i>Ongedefinieërd</i> as $\tan 2x = 0$ $x = 0^\circ$ <i>or/of</i> $\tan 2x$ is undefined if/ <i>ongedefinieërd</i> as $2x = 90^\circ$ $x = 45^\circ$	✓ $x = 0^\circ$ ✓ $x = 45^\circ$ (2)
5.5.1	$\begin{aligned} f(x) &= 2 \cos x - \sin^2 x \\ &= 2 \cos x - (1 - \cos^2 x) \\ &= 2 \cos x - 1 + \cos^2 x \\ &= \cos^2 x + 2 \cos x + 1 - 1 - 1 \\ &= (\cos x + 1)^2 - 2 \end{aligned}$	✓ identity/ <i>identiteit</i> ✓ completing the square/ <i>voltooi die vierkant</i> (2)
5.5.2	Maximum of/ <i>Maksimum van</i> $\cos x = 1$ Max. of/ <i>Maks. van</i> $(\cos x + 1)^2$ is $(1 + 1)^2 = 4$ Max. of/ <i>Maks. van</i> $(\cos x + 1)^2 - 2$ is 2	✓ Max. of/ <i>Maks. van</i> $\cos x = 1$ ✓ answer/ <i>antwoord</i> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> Answer only: Full marks Slegs antwoord: Volpunte </div> (2)
		[26]

QUESTION/VRAAG 6

6.1	$\cos(x - 30^\circ) + 2 \sin x = 0$ $\cos x \cos 30^\circ + \sin x \sin 30^\circ + 2 \sin x = 0$ $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x + 2 \sin x = 0$ $\sqrt{3} \cos x + \sin x + 4 \sin x = 0$ $5 \sin x = -\sqrt{3} \cos x$ $\tan x = -\frac{\sqrt{3}}{5}$	<p>✓ expansion of compound angle/ <i>uitbreiding van saamgestelde hoek</i></p> <p>✓ special angles/<i>spesiale hoeke</i></p> <p>✓ simplification/<i>vereenvoudiging</i></p> <p>✓ $5 \sin x = -\sqrt{3} \cos x$</p> <p>(4)</p>
6.2	$\tan x = \frac{-\sqrt{3}}{5}$ <p>ref. angle/<i>verwys. hoek</i> = 19,106 ..°</p> $x = 160,89^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x = -19,11^\circ \text{ or/of } x = 160,89^\circ$ <p>OR/OF</p> $\tan x = \frac{-\sqrt{3}}{5}$ $x = -19,11^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x = -19,11^\circ \text{ or } x = 160,89^\circ$	<p>✓ ref angle/<i>verwys. hoek</i></p> <p>✓ $x = -19,11^\circ$</p> <p>✓ $x = 160,89^\circ$</p> <p>OR/OF</p> <p>✓ $x = -19,11^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$</p> <p>✓ $x = -19,11^\circ$</p> <p>✓ $x = 160,89^\circ$</p> <p>(3)</p>
6.3.1	Amplitude = 2	<p>✓ answer/<i>antwoord</i></p> <p>(1)</p>
6.3.2		
	<p>✓ both <i>x</i>-intercepts/<i>albei x</i>-afsnitte</p> <p>✓ turning points/<i>draaipunte</i> $(-150^\circ; -1)$, $(30^\circ; 1)$, $(210^\circ; -1)$</p> <p>✓ shape/<i>vorm</i></p> <p>(3)</p>	
6.3.3	<p>(a) $x \in (-19,11^\circ; 160,89^\circ)$</p> <p>or</p> $-19,11^\circ < x < 160,89^\circ$	<p>✓ Correct intervals/<i>korrekte intervalle</i></p> <p>✓ Correct notations/<i>korrekte notasies</i></p> <p>(2)</p>
	(b) $x = 180^\circ$	<p>✓ answer</p> <p>(1)</p>
[14]		

QUESTION/VRAAG 7



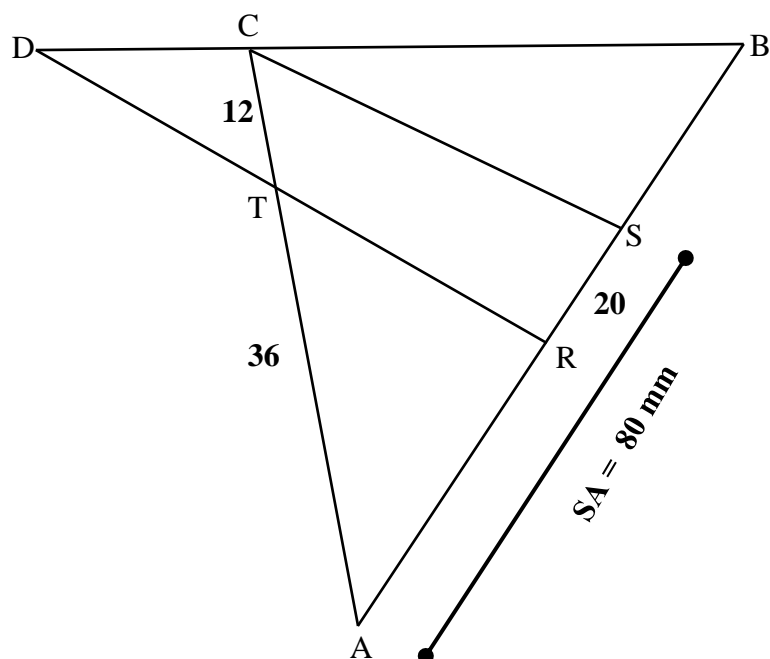
7.1	$\text{Area / Opp} = \frac{1}{2} \times k \times 2k \times \sin \hat{M}PQ$ $2k^2 \cdot \sin \theta \cos \theta = \frac{1}{2} \times k \times 2k \times \sin \hat{M}PQ$ $k^2 \cdot \sin 2\theta = k^2 \times \sin \hat{M}PQ$ $\sin 2\theta = \sin \hat{M}PQ$ $\hat{M}PQ = 2\theta$	<p>✓ subs in area rule/vervang in opp. reël</p> <p>✓ area of/van $\Delta MPQ = k^2 \times \sin \hat{M}PQ$</p> <p>✓ $2k^2 \cdot \sin \theta \cdot \cos \theta = k^2 \sin 2\theta$</p> <p style="text-align: right;">(3)</p>
7.2	$MQ^2 = k^2 + (2k)^2 - 2(k)(2k) \cos 2\theta$ $MQ^2 = k^2 + 4k^2 - 4k^2 \cos 2\theta$ $= 5k^2 - 4k^2 \cos 2\theta$ $= 5k^2 - 4k^2 (1 - 2\sin^2 \theta)$ $= 5k^2 - 4k^2 + 8k^2 \sin^2 \theta$ $= k^2 + 8k^2 \sin^2 \theta$ $= k^2 (1 + 8\sin^2 \theta)$ $MQ = k\sqrt{1 + 8\sin^2 \theta}$	<p>✓ correct subst into cosine rule/korrekte substitusie in die cos reël</p> <p>✓ $5k^2 - 4k^2 \cos 2\theta$</p> <p>✓ $\cos 2\theta = 1 - 2\sin^2 \theta$</p> <p>✓ $k^2 (1 + 8\sin^2 \theta)$</p> <p style="text-align: right;">(4)</p>
7.3	$MQ = 139,5\sqrt{1 + 8\sin^2 42^\circ}$ $MQ = 298,6045..$ $\frac{MT}{MQ} = \sin \theta$ $MT = 298,6045.. \times \sin 42^\circ$ $MT = 199,805..$ $MT \approx 200 \text{ m}$	<p>✓ value of/waarde van MQ</p> <p>✓ ratio/verhouding</p> <p>✓ $MT = 200 \text{ m}$</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>1 mark penalty for incorrect rounding/ Penaliseer met 1 punt vir verkeerde afronding</p> </div> <p style="text-align: right;">(3)</p>
[10]		

QUESTION/VRAAG 8

8.1			
8.1.1	$\hat{S}_1 = 10^\circ$	$\angle \text{centre} = 2 \times \angle \text{at circumference/midpt}$ $\angle = 2 \times \text{omtrek } \angle$	\checkmark S \checkmark R (2)
8.1.2	$\hat{R}_3 = 20^\circ$	Alt \angle 's/Verwis. \angle^e , RQ//YO	\checkmark S/R (1)
8.1.3	$\hat{P} = 150^\circ$	opposite \angle 's of cyclic quad/teenoorst. \angle^e van koordevierhoek	\checkmark S \checkmark R (2)
8.1.4	$\hat{R}_3 + \hat{R}_4 = 90^\circ$ $\hat{R}_4 = 90^\circ - 20^\circ = 70^\circ$ $\hat{S}_2 = \hat{R}_4 = 70^\circ$ OR/OF $\hat{R}_1 + \hat{R}_2 = 90^\circ$ $\hat{R}_1 = 80^\circ$ $\hat{Q}_2 = 80^\circ$ $\hat{S}_2 = 70^\circ$	radius \perp tangent/raaklyn tan chord theorem/raaklyn koord stelling OR/OF radius \perp tangent/raaklyn tan chord theorem/raaklyn koord stelling sum of \angle 's in a triangle/binne \angle^e van 'n driehoek	\checkmark R \checkmark S \checkmark S \checkmark R OR/OF \checkmark R \checkmark S \checkmark S/R \checkmark S (4)

8.2	$\hat{T}_1 = 90^\circ$ $\hat{O}_2 = 70^\circ$ $\hat{R}_4 = \hat{O}_2 = 70^\circ$ $\therefore XRY$ is a tangent/ <i>n raaklyn</i> $\hat{R}_1 + \hat{R}_2 = 90^\circ$ $\hat{T}_1 = 90^\circ$ $O\hat{R}Y = \hat{T}_1$ $\therefore XRY$ is a tangent/ <i>n raaklyne</i>	line from centre to midpoint of chord/ <i>lyn</i> <i>vanaf middelpunt tot middelpunt van koord</i> sum of \angle^s in a triangle/ <i>binne \angle^e van 'n</i> <i>driehoek</i> converse tan chord theorem/ <i>omgekeerde</i> <i>raaklyn koord stelling</i> OR/OF radius \perp tangent/ <i>raaklyn</i> line from centre to midpoint of chord/ <i>lyn</i> <i>vanaf middelpunt tot middelpunt koord</i> converse tan chord theorem/ <i>omgekeerde</i> <i>raaklyn koord stelling</i>	\checkmark S/R \checkmark S \checkmark R (3) \checkmark S \checkmark S/R \checkmark R (3)
		[12]	

QUESTION/VRAAG 9



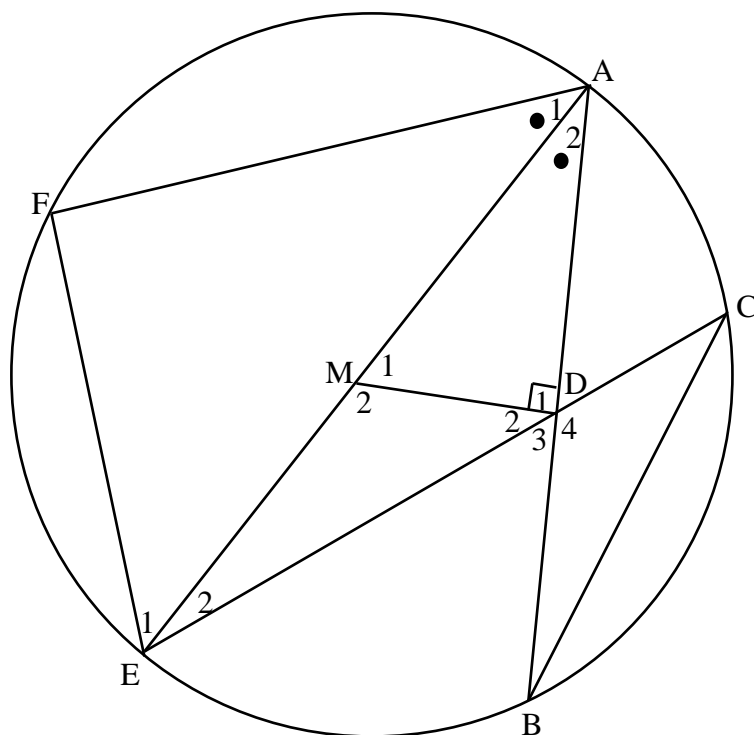
9.1	$\frac{CT}{TA} = \frac{12}{36} = \frac{1}{3}$ $\frac{SR}{RA} = \frac{20}{60} = \frac{1}{3}$ $\therefore \frac{CT}{TA} = \frac{SR}{RA}$ $\therefore CS \parallel TR$ <p style="text-align: center;">line divides the sides of Δ in proportion/<i>lyn verdeel die sye van die Δ in verhouding</i></p> <p>OR/OF</p> $\frac{SR}{SA} = \frac{20}{80} = \frac{1}{4}$ $\frac{CT}{CA} = \frac{12}{48} = \frac{1}{4}$ $\therefore \frac{CT}{CA} = \frac{SR}{SA}$ $\therefore CS \parallel TR$ <p style="text-align: center;">line divides the sides of Δ in proportion/<i>lyn verdeel die sye van die Δ in verhouding</i></p>	<p>✓S</p> <p>✓S</p> <p>✓R (3)</p> <p>✓S</p> <p>✓S</p> <p>✓R (3)</p>
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9.2	$\frac{AR}{RB} = \frac{2}{3}$ $\frac{60}{RB} = \frac{2}{3}$ $RB = 90 \text{ mm}$ $\therefore SB = 70 \text{ mm}$ $\frac{BS}{SR} = \frac{BC}{CD} \quad \text{proportion theorem/eweredigheidstelling CS TR}$ $\frac{70}{20} = \frac{2x}{\frac{1}{2}x + 1}$ $40x = 35x + 70$ $x = 14 \text{ mm}$ <p style="text-align: center;">OR/OF</p> $2k = 60$ $\therefore k = 30$ $\therefore 3k = 90$ $\frac{SR}{RB} = \frac{DC}{DB} \quad \text{prop. theorem/eweredigheidstelling CS TR}$ $\frac{20}{90} = \frac{\frac{1}{2}x + 1}{\frac{5}{2}x + 1}$ $5x + 2 = \frac{9}{2}x + 9$ $\frac{1}{2}x = 7$ $\therefore x = 14 \text{ mm}$	<p>✓S</p> <p>✓ value of/waarde van RB</p> <p>✓S ✓R</p> <p>✓ substitution/substitutie</p> <p>✓ answer/antwoord (6)</p> <p>✓ value of/waarde van k</p> <p>✓ value of/waarde van 3k</p> <p>✓S ✓R</p> <p>✓ substitution/substitutie</p> <p>✓ answer/antwoord (6)</p>
[9]		

QUESTION/VRAAG 10

10.1	<p>NB: No construction/Geen konstruksie nie 0/6</p>	
	<p>On AB, mark off AP = DE and on AC, mark off AQ = DF./Op AB, merk AP = DE of en op AC, merk AQ = DF af. Draw/Trek PQ. Proof/Bewys: In $\triangle APQ$ and/en $\triangle DEF$</p> <p>$\hat{A} = \hat{D}$ given/gegee $AQ = DF$ construction/konstruksie $AP = DE$ construction/konstruksie $\therefore \triangle APQ \cong \triangle DEF$ S\angleS</p> <p>$\hat{A}PQ = \hat{E}$ $\hat{A}PQ = \hat{B}$ [$\hat{E} = \hat{B}$]</p> <p>$\therefore PQ \parallel BC$ corresponding angles are equal/ooreenk. hoeke is gelyk</p> <p>$\frac{AB}{AP} = \frac{AC}{AQ}$ line \parallel to one side of Δ/lyn \parallel aan een sy van Δ</p> <p>$AP = DE$ and/en $AQ = DF$</p> <p>$\therefore \frac{AB}{DE} = \frac{AC}{DF}$</p>	<p>✓ construction/ konstruksie</p> <p>✓S ✓R</p> <p>✓S</p> <p>✓R</p> <p>✓S/R</p> <p>(6)</p>

10.2



<p>10.2.1</p>	<p>$\hat{F} = 90^\circ$ \angle in semi - circle/<i>halwe sirkel</i></p> <p>In $\triangle AEF$ and/en $\triangle AMD$</p> <p>$\hat{A}_1 = \hat{A}_2$ given/<i>gegee</i></p> <p>$\hat{F} = \hat{D}_1 = 90^\circ$ proved/<i>bewys</i></p> <p>$\therefore \triangle AEF \parallel \triangle AMD$ $\angle \angle \angle$</p> <p>OR/OF</p> <p>$\hat{F} = 90^\circ$ \angle in semi - circle/<i>halwe sirkel</i></p> <p>In $\triangle AEF$ and/en $\triangle AMD$</p> <p>$\hat{A}_1 = \hat{A}_2$ given/<i>gegee</i></p> <p>$\hat{F} = \hat{D}_1 = 90^\circ$ proved/<i>bewys</i></p> <p>$\hat{E}_1 = \hat{M}_1$ sum of \angle's in \triangle/<i>binne \angle e van \triangle</i></p> <p>$\therefore \triangle AEF \parallel \triangle AMD$</p>	<p>✓S/R</p> <p>✓S</p> <p>✓S</p> <p>✓R (4)</p> <p>OR/OF</p> <p>✓S/R</p> <p>✓S</p> <p>✓S</p> <p>✓R</p> <p>(4)</p>

10.2.2	$\frac{AF}{AD} = \frac{AE}{AM}$ <p>but/maar $AE = 2AM$</p> $\frac{AF}{AD} = \frac{2AM}{AM} = 2$	$\triangle AEF \parallel \triangle AMD$ \checkmark S/R \checkmark S \checkmark answer/antwoord (3)
10.2.3	<p>In $\triangle CDB$ and/en $\triangle ADE$</p> $\hat{C} = \hat{A}_2$ <p>\angles in same segment/\anglee in dieselfde segment</p> $\hat{D}_4 = \hat{E}DA$ <p>vertically opposite \angle's/regoorsaande \anglee</p> $\triangle CDB \parallel \triangle ADE$ <p>$\angle\angle\angle$</p> $\therefore \frac{CD}{AD} = \frac{DB}{DE}$ <p>but/maar $AD = DB$</p> $\frac{CD}{AD} = \frac{AD}{DE}$ <p>$\therefore AD^2 = CD \times DE$</p> <p>OR/OF</p> <p>In $\triangle CDB$ and/en $\triangle ADE$</p> $\hat{C} = \hat{A}_2$ <p>\angles in same segment/\anglee in dieselfde segment</p> $\hat{D}_4 = \hat{E}DA$ <p>vertically opposite \angle's/regoorsaande \anglee</p> $\hat{B} = \hat{E}_2$ <p>\angles in same segment/\anglee in dieselfde segment</p> $\triangle CDB \parallel \triangle ADE$ <p>$\therefore \frac{CD}{AD} = \frac{DB}{DE}$</p> <p>but/maar $AD = DB$</p> $\frac{CD}{AD} = \frac{AD}{DE}$ <p>$\therefore AD^2 = CD \times DE$</p>	\checkmark identifying \triangle s/ <i>identifiseer \trianglee</i> \checkmark S/R \checkmark S \checkmark R \checkmark S \checkmark S/R OR/OF \checkmark identifying \triangle s/ <i>identifiseer \trianglee</i> \checkmark S/R \checkmark S \checkmark R \checkmark S \checkmark S/R (6)
		[19]

TOTAL/TOTAAL: 150



**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIORSERTIFIKAAT***

GRADE/*GRAAD* 12

SEPTEMBER 2023

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 14 pages.
Hierdie nasienriglyn bestaan uit 14 bladsye.

QUESTION 1/VRAAG 1

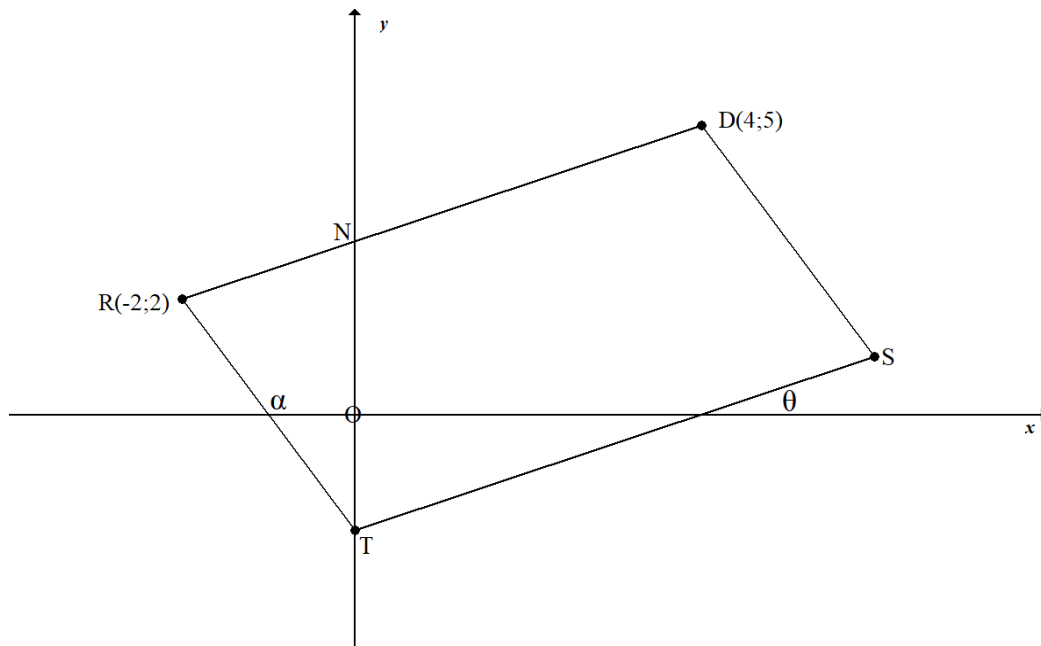
Data Set / <i>Datastel</i> : 29 27 24 31 22 19 30			
1.1.1(a)	Mean / <i>Gemiddelde</i> = $\frac{182}{7} = 26$	✓✓ mean / <i>gemiddelde</i>	(2)
1.1.1(b)	Standard deviation / <i>standaardafwyking</i> = 4,14	✓ SD / <i>standaardafwyking</i>	(1)
1.1.2	{26 – 4,14; 26 + 4,14} = {21, 86; 30, 14} 5 players / <i>spelers</i>	✓✓ calculations <i>berekeninge</i> ✓ answer / <i>antwoord</i>	(3)
1.1.3	Rugby team has the same average number of push-ups. The rugby team results were clustered around the mean because of the smaller standard deviation. <i>Rugbyspan het dieselfde gemiddelde aantal opstote. Die rugbyspan se uitslae was rondom die gemiddelde gegroepeer agv die kleiner standaardafwyking.</i>	✓✓ for any two valid comments using SD and the mean <i>Vir enige twee geldige opmerkings in gebruik van standaardafwyking en die gemiddelde</i>	(2)
1.2.1	50%	✓ answer / <i>antwoord</i>	(1)
1.2.2	Mean / <i>Gemiddelde</i> Distribution skewed to the right. (positively skewed) <i>Verspreiding is skeef na regs (positief skeef)</i>	✓ answer / <i>antwoord</i> ✓ reason / <i>rede</i>	(2)
			[11]

QUESTION 2/VRAAG 2

Third Term % <i>Derde Kw. %</i>	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final Term % <i>Finale Kw. %</i>	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71

2.1	a = 9,035 b = 0,895 $\hat{y} = 9,035 + 0,895x$	✓ for a / <i>vir a</i> ✓ for b / <i>vir b</i> ✓ for equation / <i>vir vergelyking</i>	(3)
2.2	r = 0,98	✓ answer / <i>antwoord</i>	(1)
2.3.1	$y = 9,035 + 0,895(48)$ $y \approx 52$	✓ substitution / <i>vervanging</i> ✓ answer / <i>antwoord</i>	(2)
2.3.2	correlation is very strong OR 48 is within domain of regression line. <i>korrelasie is baie sterk OF 48 is binne die gebied van die regressie-lyn.</i>	✓ answer / <i>antwoord</i>	(1)
2.4.1	50% is outside the domain of the line (data set) OR (50 ; 80) is an outlier. <i>50% is buite die gebied van die lyn (datastel) OF (50 ; 80) is 'n uitskieter</i>	✓ answer / <i>antwoord</i>	(1)
2.4.2	Increase the gradient / <i>Vermeerder die gradiënt</i>	✓ answer / <i>antwoord</i>	(1)
			[9]

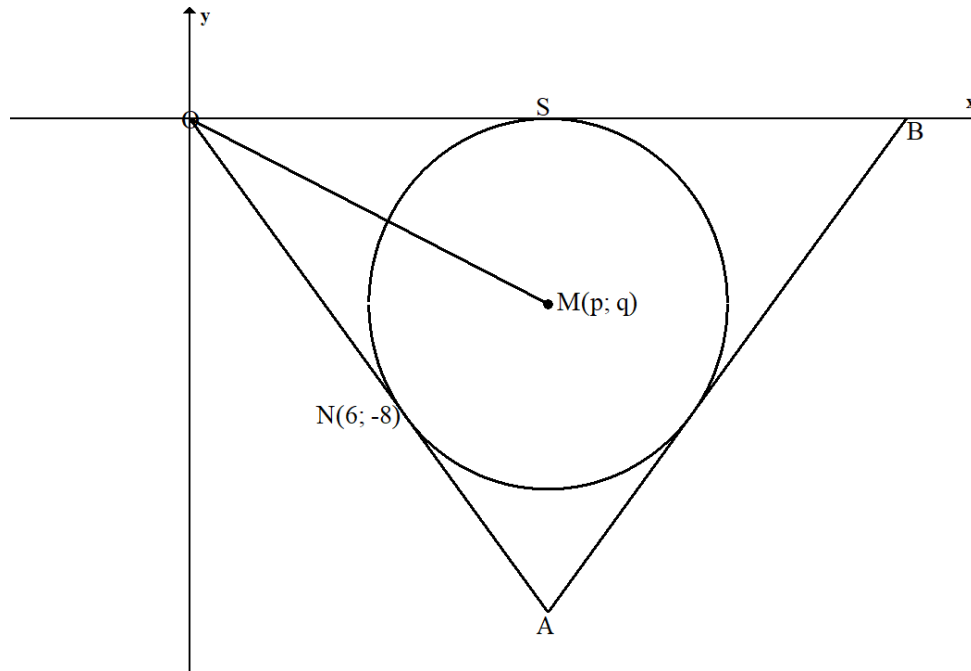
QUESTION 3 / VRAAG 3



3.1	$T(0; -2)$	✓ answer / antwoord	(1)
3.2.1	$m_{RT} = \frac{2 - (-2)}{-2 - 0} = -\frac{4}{2} = -2$	✓ substitution / vervanging ✓ answer / antwoord	(2)
3.2.2	$\tan \theta = \frac{1}{2}$ $\therefore \theta = 26,57^\circ$ $\tan \alpha = -2$ $\therefore \alpha = 116,57^\circ$ $\therefore R\hat{T}S = 116,57^\circ - 26,57^\circ = 90^\circ$ <p style="text-align: center;">OR / OF</p> $m_{TS} = \frac{1}{2}$ $m_{RT} = -2$ $m_{TS} \times m_{RT} = -1$ $R\hat{T}S = 90^\circ$	✓ for / vir $\tan \theta = \frac{1}{2}$ ✓ for / vir $\theta = 26,57^\circ$ ✓ for / vir $\tan \alpha = -2$ ✓ for / vir $\alpha = 116,57^\circ$ ✓ $R\hat{T}S = 90^\circ$ <p style="text-align: center;">OR / OF</p> ✓ $m_{TS} = \frac{1}{2}$ ✓ $m_{RT} = -2$ ✓✓ product of gradients <i>produk van gradiënte</i> ✓ $R\hat{T}S = 90^\circ$	(5)
3.3	$y = \frac{1}{2}x + c$ $2 = \frac{1}{2}(-2) + c$ $c = 3$ $\therefore y = \frac{1}{2}x + 3$	✓ gradient / gradiënt ✓ substitution of point R or D <i>vervanging van punt R of D</i> ✓ answer / antwoord	(3)

3.4	<p>RDST is a parallelogram (opposite sides parallel) RDST is 'n parallelogram (teenoorst. sye ewewydig) Midpoint of DT / Middelpunt van DT: $\left(\frac{4+0}{2}; \frac{5-2}{2}\right) = \left(2; \frac{3}{2}\right)$ Midpoint of RS is the same as midpoint of DT (diagonals bisect each other.) Middelpunt van RS is dieselfde as die middelpunt van DT (hoeklyne halveer mekaar) Midpoint of RS / Middelpunt van RS: $\left(2; \frac{3}{2}\right)$</p> <p style="text-align: center;">OR / OF</p> <p>S(6 ; 1) Midpoint of RS / Middelpunt van RS: $\left(\frac{6-2}{2}; \frac{1+2}{2}\right) = \left(2; \frac{3}{2}\right)$</p>	<p>✓ substitution in the MP formula vervanging in die MP formule ✓ S/R ✓ answer / antwoord</p> <p style="text-align: center;">OR / OF</p> <p>✓✓ S(6 ; 1) ✓ answer / antwoord</p>	(3)
3.5	<p>N(0; 3) RN = $\sqrt{2^2 + 1^2} = \sqrt{5}$ RT = $\sqrt{2^2 + 4^2} = \sqrt{20}$</p> <p>Area / Oppervlakte = $\frac{1}{2} \times \sqrt{20} \times \sqrt{5}$ = 5 square units / eenhede²</p> <p style="text-align: center;">OR / OF</p> <p>TN = 5 units / eenhede Height / Hoogte = 2 units / eenhede Area / Oppervlakte = $\frac{1}{2} \times 5 \times 2$ Area / Oppervlakte = 5 square units / eenhede²</p>	<p>✓ coordinates of N koördinate van N ✓ for / vir RN ✓ for / vir RT</p> <p>✓ for the answer / vir die antwoord</p> <p style="text-align: center;">OR / OF</p> <p>✓ TN = 5 units / eenhede ✓ Height/hoogte = 2 units / eenhede ✓ sub. into formula vervanging in formule ✓ answer / antwoord</p>	(4)
			[18]

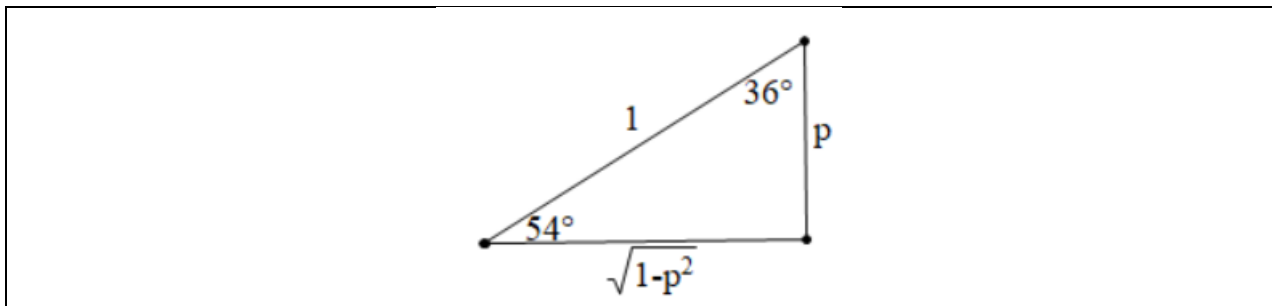
QUESTION 4 / VRAAG 4



4.1.1	$ON = \sqrt{(6 - 0)^2 + (-8)^2}$ $= \sqrt{100} = 10 \text{ units / eenhede}$	✓ substitution in correct formula <i>vervanging in korrekte formule</i> ✓ answer / antwoord	(2)
4.1.2	$ON = OS$ (tangents from the same point) <i>(raaklyne vanaf dieselfde punt)</i> $\therefore p = 10 \text{ units / eenhede}$	✓ S and/en R ✓ answer / antwoord	(2)
4.1.3	$ON \perp NM$ (tan – radius) / <i>(raaklyn – radius)</i> $m_{ON} = \frac{-8}{6} = \frac{-4}{3}$ $m_{NM} = \frac{3}{4}$	✓ S and/en R ✓ gradient of ON / <i>gradiënt van ON</i> ✓ gradient of NM <i>gradiënt van NM</i>	(3)
4.1.4	$m_{NM} = \frac{q+8}{10-6} = \frac{3}{4}$ $\frac{q+8}{4} = \frac{3}{4}$ $q = -5$	✓ for subs and equating <i>vir vervanging en gelyk stel</i> ✓ answer / antwoord	(2)
4.2	$MS = r = 5 \text{ units / eenhede}$ $(x - 10)^2 + (y + 5)^2 = 25$	✓ radius / radius ✓ centre sub/ <i>vervang middelpunt</i> ✓ answer / antwoord	(3)
4.3	$k = 5$ OR/OF $k = 15$	✓ $k = 5$ OR/OF ✓ $k = 15$	(2)

4.4	<p>Coordinates of the point directly opposite N is C. <i>Koördinate van die punt regoor N is C.</i></p> $C\left(\frac{x+6}{2} = 10; \frac{y-8}{2} = -5\right)$ $C(14; -2)$ <p>Equation of the tangent at C: <i>Vergelyking van die raaklyn by C:</i></p> $y + 2 = -\frac{4}{3}(x - 14)$ $y = -\frac{4}{3}x + \frac{50}{3}$ $\therefore 0 < t < \frac{50}{3}$	<p>✓ formula and sub / <i>formule en vervanging</i></p> <p>✓ for x-coordinate <i>vir x-koördinaat</i></p> <p>✓ for y-coordinate <i>vir y-koördinaat</i></p> <p>✓ substitution / vervanging</p> <p>✓ for the answer / <i>vir die antwoord</i></p> <p>✓ for the value of t. <i>vir die waarde van t</i></p>	(6)
4.5	<p>They will not touch. The new circle is the old circle shifted up by 11.</p> <p><i>Hulle sal nie raak nie.</i> <i>Die nuwe sirkel is die ou sirkel 11 eenhede opwaarts geskuif.</i></p>	<p>✓ answer / <i>antwoord</i></p> <p>✓ any valid reason / <i>enige geldige antwoord</i></p>	(2)
			[22]

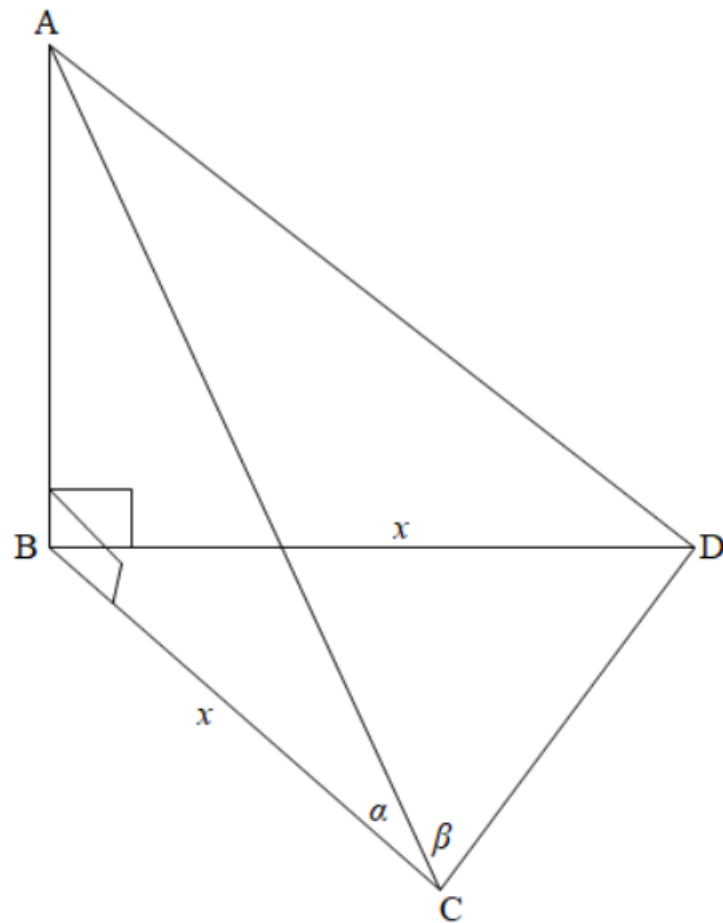
QUESTION 5/VRAAG 5



5.1.1	$\begin{aligned} &\sin 594^\circ \\ &= \sin 234^\circ \\ &= -\sin 54^\circ \\ &= -p \end{aligned}$	$\begin{aligned} &\checkmark -\sin 54^\circ \\ &\checkmark -p \end{aligned}$	(2)
5.1.2	$\begin{aligned} &\cos 36^\circ \\ &= p \end{aligned}$	$\checkmark\checkmark \text{ answer / antwoord}$	(2)
5.1.3	$\begin{aligned} &\cos 18^\circ \\ &= \sin 72^\circ \\ &= \sin 2(36^\circ) \\ &= 2 \sin 36^\circ \cos 36^\circ \\ &= 2 \left(\frac{\sqrt{1-p^2}}{1} \right) \left(\frac{p}{1} \right) \\ &= 2p(\sqrt{1-p^2}) \end{aligned}$	$\begin{aligned} &\checkmark \text{ for / vir } \sin 72^\circ \\ &\checkmark \text{ for / vir } \sin 2(36^\circ) \\ &\checkmark \text{ for / vir } 2 \sin 36^\circ \cos 36^\circ \\ &\checkmark \text{ for / vir } 2p(\sqrt{1-p^2}) \end{aligned}$	(4)
5.2	$\begin{aligned} &\frac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 410^\circ + \cos(-\theta)} \\ &= \frac{\cos(90^\circ + 50^\circ) - \cos \theta}{\sin 50^\circ + \cos \theta} \\ &= \frac{-\sin 50^\circ - \cos \theta}{\sin 50^\circ + \cos \theta} \\ &= \frac{-(\sin 50^\circ + \cos \theta)}{(\sin 50^\circ + \cos \theta)} \\ &= -1 \end{aligned}$	$\begin{aligned} &\checkmark -\sin 50^\circ \quad \checkmark \cos \theta \\ &\checkmark \sin 50^\circ \quad \checkmark \cos \theta \\ &\checkmark \text{ for the common factor} \\ &\quad \text{vir die gemene faktor} \\ &\checkmark \text{ for the answer} \\ &\quad \text{vir die antwoord} \end{aligned}$	(6)

5.3	$\cos(x + 65^\circ) \cdot \cos(x + 20^\circ) - \sin(x + 245^\circ) \cdot \sin(x + 20^\circ)$ $= \cos(x + 65^\circ) \cdot \cos(x + 20^\circ) + \sin(x + 65^\circ) \cdot \sin(x + 20^\circ)$ $= \cos[(x + 65^\circ) - (x + 20^\circ)]$ $= \cos 45^\circ$ $= \frac{1}{\sqrt{2}}$	<p>✓ reduction / <i>reduksie</i></p> <p>✓ compound angle <i>saamgestelde hoek</i></p> <p>✓ $\cos 45^\circ$</p> <p>✓ answer / <i>antwoord</i></p>	(4)
5.4	$\cos^2 x - \sin^2 x = \frac{1}{2}$ $\cos 2x = \frac{1}{2}$ $2x = 60^\circ + 360^\circ \cdot k \text{ or/of } 2x = 300^\circ + 360^\circ \cdot k$ $x = 30^\circ + 180^\circ \cdot k \text{ or/of } x = 150^\circ + 180^\circ \cdot k \quad k \in \mathbb{Z}$ <p style="text-align: center;">OR / OF</p> $2 \cos^2 x - 2 \sin^2 x = 1$ $2 \cos^2 x - 2 \sin^2 x = \sin^2 x + \cos^2 x$ $3 \sin^2 x = \cos^2 x$ $\tan^2 x = \frac{1}{3}$ $\tan x = \frac{1}{\sqrt{3}}$ $x = 30^\circ \text{ (reference angle / } \textit{verwysingshoek})$ $x = 30^\circ + 180^\circ \cdot k, k \in \mathbb{Z}$	<p>✓ $\cos 2x = \frac{1}{2}$</p> <p>✓ for / <i>vir</i> $2x$ in both quadrants / <i>in beide kwadrate</i></p> <p>✓ $x = 30^\circ + 180^\circ \cdot k$</p> <p>✓ $x = 150^\circ + 180^\circ \cdot k$</p> <p style="text-align: center;">OR / OF</p> <p>✓ multiplying by 2 and using identity / <i>vermenigvuldig met 2 en gebruik van identiteit</i></p> <p>✓ $3 \sin^2 x = \cos^2 x$</p> <p>✓ $\tan x = \frac{1}{\sqrt{3}}$</p> <p>✓ answer / <i>antwoord</i></p>	(4)
5.5.1	$\text{LHS} = \frac{\sin 2\theta \cdot \tan \theta}{\cos 2\theta + 1}$ $= \frac{2 \sin \theta \cos \theta \cdot \frac{\sin \theta}{\cos \theta}}{2 \cos^2 \theta - 1 + 1}$ $= \frac{2 \sin^2 \theta}{2 \cos^2 \theta}$ $= \tan^2 \theta$	<p>✓ $2 \sin \theta \cos \theta$</p> <p>✓ $\frac{\sin \theta}{\cos \theta}$</p> <p>✓ $2 \cos^2 \theta - 1$</p> <p>✓ $\frac{2 \sin^2 \theta}{2 \cos^2 \theta}$</p>	(4)
5.5.2	$\cos 2\theta + 1 = 0$ $\cos 2\theta = -1$ $2\theta = 180^\circ$ $\theta = 90^\circ$	<p>✓ $\cos 2\theta + 1 = 0$</p> <p>✓ $\cos 2\theta = -1$</p> <p>✓ $2\theta = 180^\circ$</p> <p>✓ $\theta = 90^\circ$</p>	(4)
			[30]

QUESTION 6 / VRAAG 6



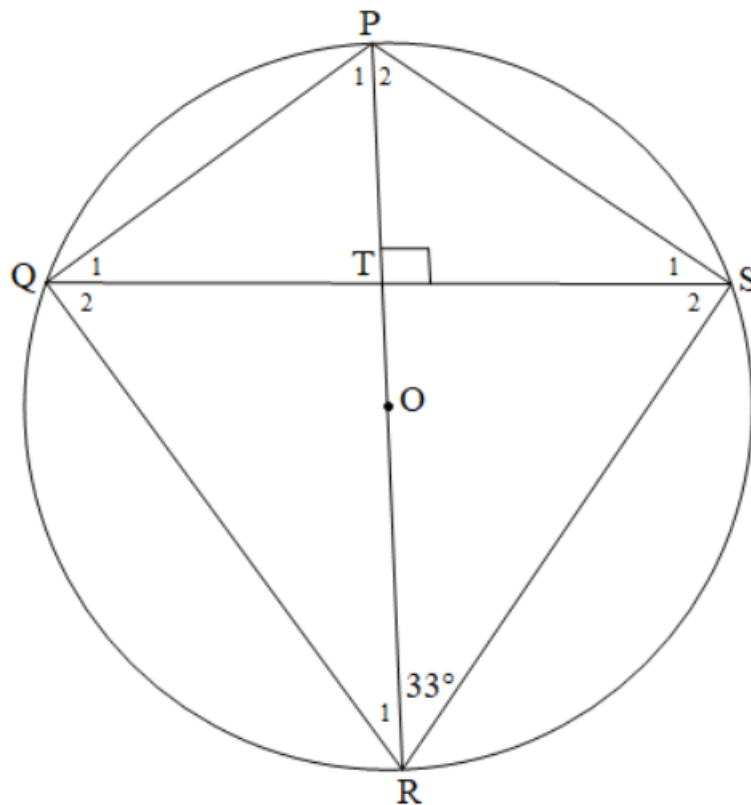
6.1	$\Delta ABC \equiv \Delta ABD$ (SAS) / (SHS)	✓ S and/en R	(1)
6.2	$\cos \alpha = \frac{x}{AC}$ $AC = \frac{x}{\cos \alpha}$	✓ S ✓ answer / antwoord	(2)
6.3	$CD^2 = \left(\frac{x}{\cos \alpha}\right)^2 + \left(\frac{x}{\cos \alpha}\right)^2 - 2\left(\frac{x}{\cos \alpha}\right)\left(\frac{x}{\cos \alpha}\right)\cos(180^\circ - 2\beta)$ $CD^2 = \frac{x^2}{\cos^2 \alpha} + \frac{x^2}{\cos^2 \alpha} + 2\left(\frac{x^2}{\cos^2 \alpha}\right)\cos 2\beta$ $CD^2 = \frac{2x^2}{\cos^2 \alpha}(1 + \cos 2\beta)$ $CD^2 = \frac{2x^2}{\cos^2 \alpha}(1 + (1 - 2\sin^2 \beta))$ $CD^2 = \frac{2x^2}{\cos^2 \alpha} \times 2\cos^2 \beta$ $CD^2 = \frac{4x^2 \cos^2 \beta}{\cos^2 \alpha}$ $\therefore CD = \frac{2x \cos \beta}{\cos \alpha}$ <p style="text-align: center;">OR / OF</p>	✓ cos rule / cos-reël ✓ simplification vereenvoudiging ✓ double angles expansion dubbelhoeke uitbreiding ✓ simplification vereenvoudiging	<p style="text-align: center;">OR / OF</p>

	$\widehat{ADB} = \beta$ $\widehat{CAD} = 180^\circ - 2\beta$ $\frac{CD}{\sin(180^\circ - 2\beta)} = \frac{AC}{\sin \beta}$ $CD = \frac{AC \sin 2\beta}{\sin \beta}$ $CD = \frac{x \cdot 2 \sin \beta \cos \beta}{\cos \alpha \sin \beta}$ $CD = \frac{2x \cos \beta}{\cos \alpha}$	✓ for /vir $\widehat{ADB} = \beta$ and/en $\widehat{CAD} = 180^\circ - 2\beta$ ✓ use of sin rule <i>gebruik van sinus-reël</i> ✓ substitution of AC <i>vervanging van AC</i> ✓ simplification / <i>vereenvoudiging</i>	(4)
6.4	$CD = \frac{2x \cos \beta}{\cos \alpha}$ $CD = \frac{2(25) \cos 65,62^\circ}{\cos 30^\circ}$ $CD = 23,83 \text{ cm}$	✓ substitution / <i>vereenvoudiging</i> ✓ answer / <i>antwoord</i>	(2)
			[9]

QUESTION 7 / VRAAG 7

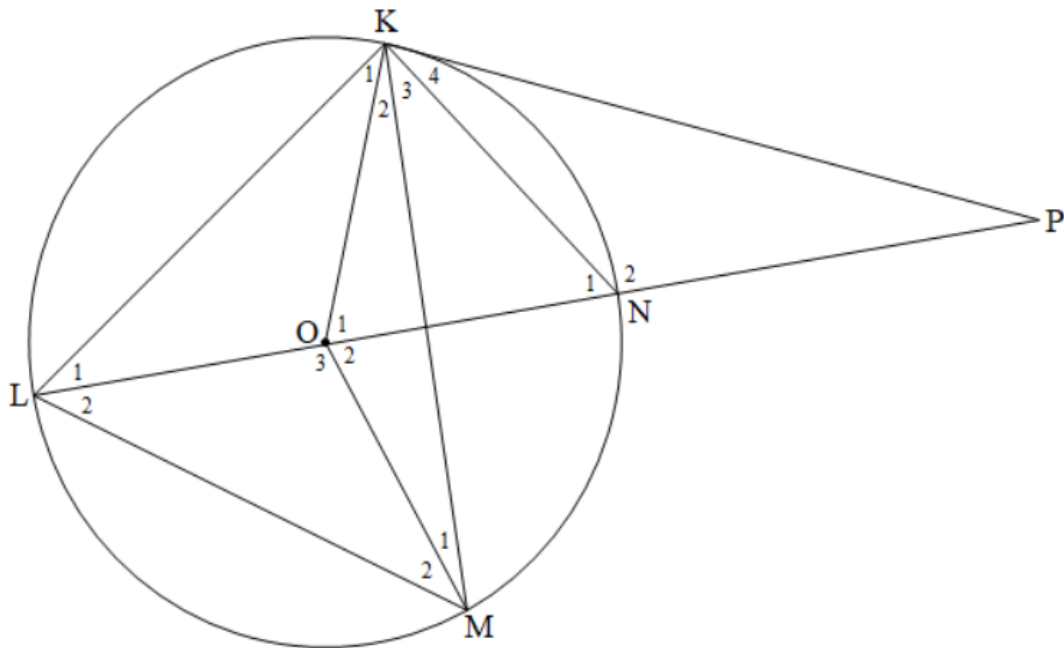
7.1	$f(180^\circ) = -0,71$ \therefore Range: <i>Terrein: Waardeversameling:</i> $-0,71 \leq y \leq 1$ OR / OF $[-0,71; 1]$	✓ $f(180^\circ)$ ✓ answer / <i>antwoord</i>	(2)
7.2	<p>✓ intercepts / <i>afsnitte</i> ✓ shape / <i>vorm</i> ✓ turning points / <i>draaipunte</i></p>		(3)
7.3	Period / <i>Periode</i> = 180°	✓ answer / <i>antwoord</i>	(1)
7.4	$-45^\circ < x < 45^\circ$	✓✓ answer / <i>antwoord</i>	(2)
7.5	$x = -45^\circ$ or / <i>of</i> $x = 135^\circ$	✓ $x = -45^\circ$ ✓ $x = 135^\circ$	(2)
7.6	$g(x) = \cos(x + 15^\circ)$	✓ answer / <i>antwoord</i>	(1)
			[11]

QUESTION 8 / VRAAG 8



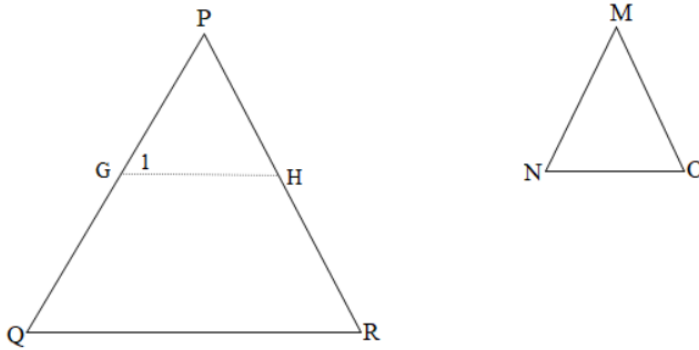
8.1.1	$\hat{Q}_1 = 33^\circ$ (\angle s in the same segment) (\angle e in dieselfde segment)	✓ S ✓ R	
	$\hat{P}_1 = 57^\circ$ (\angle s of a triangle) / (\angle e van 'n driehoek)	✓ S and/en R	(3)
8.1.2	$\hat{Q} = 90^\circ$ (\angle subtended by the diameter) (\angle onderspan deur middellyn)	✓ S and/en R	
	$\hat{Q}_2 = 57^\circ$ (complementary \angle s / \angle s of a triangle) (komplementêre \angle e / \angle e van 'n driehoek)	✓ S and/en R	(2)
8.2	$QT = TS = 8 \text{ cm}$ (line from centre perp to chord) (lyn vanaf middelpunt loodreg op koord)	✓ S and/en R	
	$OQ = OS = 10 \text{ cm}$ (radii) / (radiusse)	✓ S and/en R	
	$OQ^2 = TO^2 + QT^2$ (Pythagoras) / (Pythagoras)	✓ S and/en R	
	$10^2 = OT^2 + 8^2$		
	$TO = \sqrt{100 - 64} = 6 \text{ cm}$	✓ answer / antwoord	(4)
			[9]

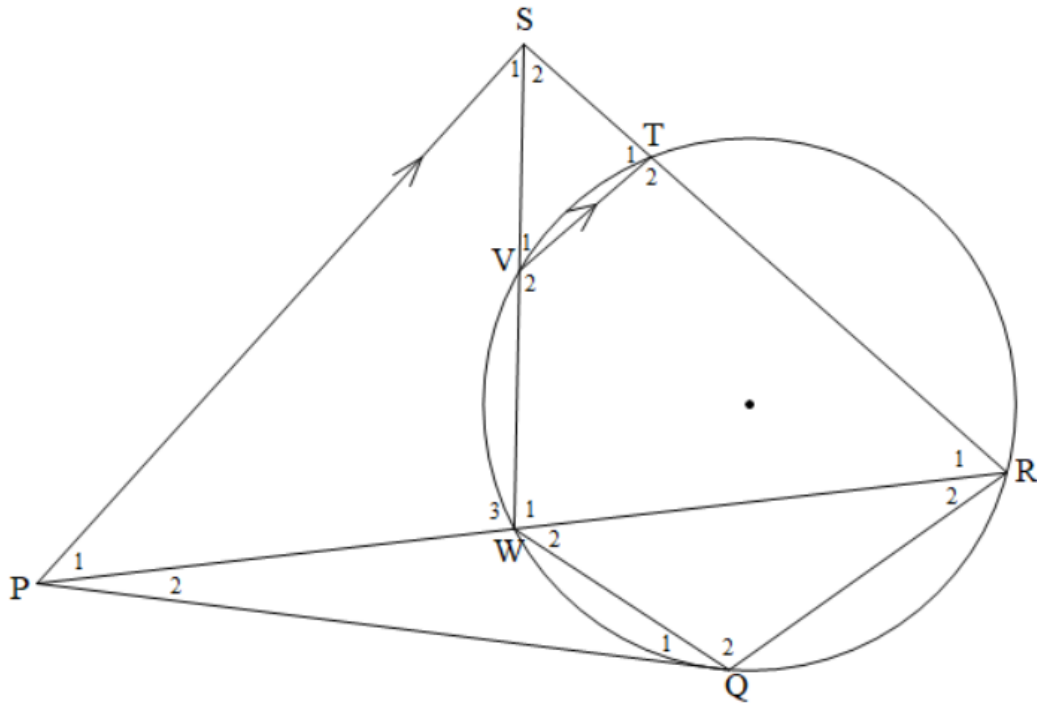
QUESTION 9 / VRAAG 9



9.1	$\hat{L}\hat{K}N = 90^\circ$ $\hat{O}\hat{K}P = 90^\circ$	✓ answer / antwoord ✓ answer / antwoord	(2)
9.2.1	$\hat{L}_1 = x$ (tan – chord theorem) / (raaklyn-koord stelling)	✓ S ✓ R	(2)
9.2.2	$\hat{K}_1 = x$ (\angle s opp. = sides) / (\angle e teenoor = sye)	✓ S ✓ R	(2)
9.2.3	In ΔLKP ; $\hat{L}\hat{K}P = 90^\circ + x$ and/en $\hat{L}_1 = x$ $\therefore \hat{P} = 90^\circ - 2x$ (\angle s of a triangle) (\angle e van 'n driehoek) OR / OF $\hat{N}_2 = 90^\circ + x$ (ext \angle of Δ) / (buite \angle van Δ) $\therefore \hat{P} = 90^\circ - 2x$ (\angle s of a triangle) (\angle e van 'n driehoek)	✓ S ✓ R OR / OF ✓ S/R ✓ S/R	(2)
9.3	$\hat{O}\hat{M}P = 90^\circ$ (tan \perp rad) / (raaklyn \perp radius) $\hat{O}\hat{K}P = 90^\circ$ (proven) / (bewys) \therefore KOMP is a c.q. (opp. \angle s suppl.) KOMP is 'n kv (teenoorst. \angle e supplementêr)	✓ S & R ✓ S ✓ R	(3)
			[11]

QUESTION 10 / VRAAG 10

<p>10.1</p>	 <p>On PQ, mark PG = MN and on PR, mark off PH = MO Join GH In $\triangle PGH$ and $\triangle MNO$ (1) PG = MN (construction) (2) $\hat{P} = \hat{M}$ (given) (3) PH = MO (construction) $\therefore \triangle PGH \equiv \triangle MNO$ (SAS) $\therefore \hat{G}_1 = \hat{N}$ (Congruency) But $\hat{Q} = \hat{N}$ (given) $\therefore \hat{G}_1 = \hat{Q}$ $\therefore GH \parallel QR$ (corresponding angles formed =) $\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (prop.int; $GH \parallel QR$) But PG = MN and PH = MO $\therefore \frac{MN}{PQ} = \frac{MO}{PR}$</p>	<p>✓ construction</p> <p>✓ congruency proof</p> <p>✓ S ✓ R</p> <p>✓ S and/en R</p> <p>✓ S</p>	
	<p>Afrikaans</p> <p>Op PQ, merk af PG = MN en op PR, merk af PH = MO Verbind GH In $\triangle PGH$ en $\triangle MNO$ (1) PG = MN (konstruksie) (2) $\hat{P} = \hat{M}$ (gegee) (3) PH = MO (konstruksie) $\therefore \triangle PGH \equiv \triangle MNO$ (SHS) $\therefore \hat{G}_1 = \hat{N}$ (Kongruensie) Maar, $\hat{Q} = \hat{N}$ (gegee) $\therefore \hat{G}_1 = \hat{Q}$ $\therefore GH \parallel QR$ (ooreenkomstige hoeke gevorm =) $\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (eweredigheid; $GH \parallel QR$) Maar, PG = MN en PH = MO $\therefore \frac{MN}{PQ} = \frac{MO}{PR}$</p>	<p>✓ konstruksie</p> <p>✓ kongruensie bewys</p> <p>✓ S ✓ R</p> <p>✓ S and/en R</p> <p>✓ S</p>	<p>(6)</p>



10.2.1	$\hat{S}_1 = \hat{V}_1$ (alt \angle s; $PS \parallel VT$) / (<i>verw. \anglee ; $PS \parallel VT$</i>) $\hat{V}_1 = \hat{R}_1$ (ext. \angle of cq $RTVW$) / (<i>buite \angle van kv $RTVW$</i>) $\therefore \hat{S}_1 = \hat{R}_1$	✓ S and/en R ✓ S ✓ R	(3)
10.2.2	In $\triangle PWS$ and/en $\triangle PSR$ (1) \hat{P}_1 is common / <i>is gemeen</i> (2) $\hat{S}_1 = \hat{R}_1$ (proven) / (<i>bewys</i>) (3) 3 rd angle of a triangle / (<i>3^{de} hoek van driehoek</i>) $\triangle PWS \parallel \triangle PSR$ (AAA) / (HHH) / ($\angle \angle \angle$)	✓ S ✓ R ✓ S and/en R	(3)
10.2.3	In $\triangle PQW$ and/en $\triangle PRQ$ (1) $\hat{Q}_1 = \hat{R}_2$ (tan – chord theorem) / (<i>raaklyn – koord stelling</i>) (2) \hat{P}_2 is common / <i>is gemeen</i> (3) 3 rd angle of a triangle / (<i>3^{de} hoek van driehoek</i>) $\triangle PQW \parallel \triangle PRQ$ (AAA) $\therefore \frac{PQ}{PR} = \frac{PW}{PQ}$ (similarity) / (<i>gelykvormig</i>) $\therefore PQ^2 = PW \cdot PR$	✓ S and/en R ✓ S ✓ S and/en R ✓ S ✓ S	(5)
10.2.4	From / <i>Vanaf</i> 10.2.1 $\frac{PW}{PS} = \frac{PS}{SR}$ ($\triangle PWS \parallel \triangle PSR$) $\therefore PS^2 = PW \cdot PR$ From / <i>Vanaf</i> 10.2.2 $PQ^2 = PW \cdot PR$ $\therefore PQ = PS$	✓ S ✓ S ✓	(3)
			[20]

TOTAL / TOTAAL: 150